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PRINCIPLES OF JAMMING AND ELECTRONIC RECONNAISSANCE

by

S. A. Vakin and L. N. Shustov

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PRINCIPLES OF JAMMING AND ELECTRONIC RECONNAISSANCE

By: S. A. Vakin and L. N. Shustov

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ABSTRACT

(U) The book is the first work in which with the using of contempory mathematical methods there is systematically expounded the theory of jamming and electronic reconnaissance. In a considerable part it is based on the original investigations of the authors. Information, power and operational-tactical criteria of the effectiveness of means and methods of the creation of interference are examined. Methods of the estimate of information loss, applied by means of active jamming to radar stations operating in scanning conditions, are described. Different forms of active jamming on the channel of angular tracking of radar (monopulse and with conical scanning) are investigated. Peculiarities of functioning radar systems of automatic tracking in the direction with the action of special interference signals are examined. Methods of the creation of active jamming to systems of automatic range and speed tracking are discussed. An estimate of different forms of interference signals to radio links of communication and command control is given. Calculation relationships are derived which allow estimating the necessary quantity of dipole reflectors for the suppression of radar of different assignment, including and pulse-coherent radar. Principles of the application of radar traps in different links of control system by means of antiaircraft defense are discussed.

TABLE OF CONTENTS

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* <u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

ANNOTATION

The book is the first work in which with the using of contemporary mathematical methods there is systematically expounded the theory of jamming and electronic reconnaissance. In a considerable part it is based on the original investigations of the authors.

Information, power and operational-tactical criteria of the effectiveness of means and methods of the creation of interference are examined. Methods of the estimate of information loss, applied by means of active jamming to radar stations operating in scanning conditions, are described.

Different forms of active jamming on the channel of angular tracking of radar (monopulse and with conical scanning) are investigated. Peculiarities of functioning radar systems of automatic tracking in the direction with the action of special interference signals are examined.

Methods of the creation of active jamming to systems of automatic range and speed tracking are discussed. An estimate of different forms of interference signals to radio links of communication and command control is given.

Calculation relationships are derived which allow estimating the necessary quantity of dipole reflectors for the suppression of radar of different assignment, including and pulse-coherent radar. Principles of the application of radar traps in different links of control system by means of antiaircraft defense are discussed. Methods of increasing the effectiveness of the area of scattering and also methods of jamming founded on the change in electrical properties of the medium and radar observation of targets are shown.

The problem of electronic intelligence is discussed on the basis of the queueing theory. There are given different circuits of devices of the determination and memorization of frequency and also the determination of the bearing and position of the electronic equipment.

The book is intended for a wide range of specialists engaged in problems of the development, exploitation and application of electronic equipment. It will be useful to teachers and students of electronic higher educational institutions and departments. The book has 223 figures and 112 names in the bibliography.

PREFACE

Individual problems in jamming [RPD] and electronic reconnaissance [RTR] were examined earlier only in journal articles and also in a number of pamphlets.

The attempt to give a systematic account of the basic scientific and technical problems of this important field of applied electronics is natural.

The development of contemporary methods of investigation, such as the theory of decisions, the theory of games and queueing theory and development of the theory of radar and radio control permitted the authors to systematize works published up to the present time and examine as far as possible well-known methods of RPD and RTR from single positions.

In considerable part the book contains data of original investigations of authors and also theoretical generalizations of published materials.

In the beginning of the book information and operational-tactical criteria of RPD effectiveness are discussed. In subsequent chapters possible methods of RPD to different forms of electronic means are examined. The main attention is given to interferences to radar operating both in conditions of scanning and in conditions of automatic tracking.

The last chapter is devoted to electronic reconnaissance. As a theoretical basis for RTR the queueing theory is used.

The book is intended on readers having a mathematical preparation from a higher technical educational institution. Used in a number of cases are new mathematical methods (games theory, queueing theory, theory of decisions and some divisions of the information theory) are additionally explained.

Chapters 1, 2, and 3 (with the exception of 1.9, 2.5, 2.6, 2.7, 3.1, 3.2, and 3.6), and also 4.5, 4.6, 5.4, 7.3, 7.7, 9.2, and 10.2 were written by S. A. Vakin, Chapter 4 (with the exception of 4.1, 4.5, and 4.6) and also 3.2, 3.6, 6.3, and 6.4 were written by L. N. Shustov. The remaining sections were written by the authors jointly.

The authors express their deep gratitude to M. P. Bobnev, B. D. Sergiyevskiy, V. T. Borovik, V. P. Malaychuk, S. S. Romanov, and B. V. Abramom for their critical remarks and councils promoting the improvement of the book.

The necessity to consider the wide range of questions, naturally, could not fail to lead to an irregularity in the depth of the study of the separate divisions. The authors will gratefully accept critical remarks on the substance of the work.

INTRODUCTION

At present electronic means [RES] comprises the basis of systems of control of troops and weapons in all forms of armed forces of contemporary states. In the most characteristic form this position appears in antiaircraft [PVO] and antimissile defense [PRO].

The development of electronics made possible the entering into armament of PVO of controlled weapons of destruction - aircraft and antiaircraft controlled rocket weaponry ([AURO] and [ZURO]) which to a great degree increased the probability of the defeating the enemy aircraft by one rocket.

If in the Second World War to destroy one aircraft it was required to use on the average of up to 500-600 shells of barrel antiaircraft artillery [ZA], then at present for aircraft destruction 1-2 antiaircraft guided missiles [ZUR] are sufficient.

At the same time the electronic facilities are one of the most vulnerable sections of the PVO system since they are detected by the radiation and their operation can be jammed, i.e., jamming by electronic methods.

The dialectics of combating measures and countermeasures, naturally, led to the appearance and development of methods of surmounting by the aviation of the PVO of the enemy founded on the application of jamming the operation of electronic means of the system of the PVO.

At present jamming is one of the important forms of the security of combat actions of aviation in the surmounting of PVO.

Jamming is widely applied in other branches of the military. However, the purposes, methods, and means of RPD in different forms of armed forces and branches of service can be different.

Thus, for example, the purpose of RPD, carried out in the interests of the cover of objects in the system of PVO and PRO can be the suppression of radar bombsights, means of radio navigation, means of control of air-to-surface and surface-to-surface rockets, and the realization of radar jamming camouflage of covered objects.

In land forces RPD can include the suppression of means of communication of the enemy in tactical and operational links, its electronic reconnaissance stations, means of control of surface-to-surface and air-to-surface rockets and antiradar camouflage of objects and others.

Electronic reconnaissance as a method of obtaining information on radar means of the enemy is the most important component part of RPD. Without electronic reconnaissance it is impossible to organize effective jamming.

Furthermore, electronic reconnaissance has an independent importance in being a component part of air or combined arms reconnaissance. In the book the main attention is given to methods and means of jamming and electronic reconnaissance intended for the application by their aviation for the surmounting of PVO of the enemy. Problems of RPD, realizable in the interests of PVO are partially examined.

The first information on the deliberate creation of radio interference pertains to the Russian-Japanese War. The commander of the Russian cruiser "Ural" proposed to the commanding squadron Vice Admiral Z. P. Rozhestvenskiy to suppress by radiation of the onboard radio station the radio link of the Japanese reconnaissance cruisers which followed at a short distance from the Russian squadron and transmitted information about its movement. However Z. P.

Rozhestvenskiy did not resolve this and thereby permitted scouts of the enemy to transmit freely to the highest command of the Japanese fleet information about the combat formation and coordinates of ships of his squadron.

In the course of the Tsushima Battle forward commanders of individual ships by their own initiative applied interference of radio communications. Thus, for example, interference was created by the cruiser "Izumrud" and torpedo boat "Gromkiy."

During the period of the First World War radio reconnaissance and the monitoring of conversations by radio found widespread use. However, the wide development of special equipment of radio interference and radio reconnaissance started to come about only during the Second World War. In this period there was already being carried out a systematic and mass application by the English and Americans of radio interference for the purpose of the suppression of radar of the PVO of fascist Germany. In 1943 the English used passive interference (metallized strips) against German gun-laying radars. In the same year on the aircraft of Great Britain there was installed a noise jammer to gun-laying radar [SON] "Würzburg." The creation of interferences was facilitated by the standardization of the German radar, which amounted by then to about 5000.

The majority of radio engineers of Germany during 1944-1945 were engaged in the development of attachments to radar of the gun-laying "Würzburg" for combating passive interferences created by means of ejection from aircraft of metallized paper strips. There was developed an attachment of alternating compensation. However, this attachment did not provide protection with great density of interferences.

Radio misinformation (the transmission of false instructions in the guidance system of destroyers) was widely applied.

In contemporary conditions of high saturation of PVO and PRO by electronic means and the immeasurably increasing strike shock force of rockets and aircraft, the role and importance of RPD are especially

great. Already at present without the application of jamming, the probability of surmounting the PVO of the enemy by a single aircraft is very low.

Potential possibilities of electronic means are such that in principle it is possible to expect the mass introduction of systems providing just as successful interception and intercontinental ballistic missiles. Thus, there is basis to recognize as real the creation, with the help of electronic means and controlled weapons of destruction, of a reliable "dynamic shield" covering the territory of the country or its important objects from strikes of aircraft and rockets. The surmounting of this shield by means of application of only weapons of destruction for destroying its electronic links in principle is not a radical method of solving the problem, inasmuch as each such weapon of destruction can be revealed and destroyed as any other means of air attack. The basic method of surmounting the "dynamic shield" is to disrupt the control system by means of changing the quantity of information circulating both in its different links and in the entire control system as a whole. This can be achieved, mainly, with the help of jamming and also a maneuver combined with RPD.

In virtue of the logic of armed combat development of the means and methods of jamming produced by counterjamming, into the problem of which enters the development of methods and lowering the effectiveness of RPD, which provides the possibility of obtaining information with the help of electronic means under conditions of RPD and the organization and application of means of RPD hampering the enemy.

The combating of methods of jamming and counterjamming comprises two sides of a conflicting situation, which is sometimes called radio war. A characteristic peculiarity of radio war is its high dynamicism which is caused by a strong dependence of methods of jamming and counterjamming on countermeasures of the enemy. Success in radio war is achieved by superiority over the enemy in the quantity and quality of radio electronic technology and skill of its combat application.

Thus, in the contemporary stage of development of methods of armed combat, electronic means have become weapons in the literal meaning of the word – weapon of radio war.

The determining character of the influence of jamming on the effectiveness of combat actions of aviation requires in the examining of concrete methods and means of RPD an appraisal of the change in combat effectiveness generated by them. At the same time, a correct solution to operational and tactical problems in contemporary conditions can be given only by taking into account both jamming and counterjamming.

Designations for Chapter 1

РПД -	RPD -	jamming
ПВО -	PVO -	antiaircraft defense
РЭС -	RES -	electronic systems
ИА -	IA -	fighter aircraft
ТВД -	TO -	theater of operations
КРУ -	KRU -	command radio control link
ЗУР -	ZUR -	antiaircraft guided missiles
СРП -	SRP -	computer
ГП -	GP -	gyrostabilized platform
АЦ -	ATs -	target
АСН -	ASN -	automatic direction tracking
СП -	SP -	actuator
СД -	SD -	data removal device
СГ -	SG -	rate gyroscope
ЭПР -	ESA -	effective scattering area

Designations for Chapter 2

РЛС	-	radar (when appearing as a subscript)		
ПС	-	PS	-	covered aircraft
ПП	-	PP	-	jammer
РП	-	RP	-	reconnaissance receiver
СЗЧ	-	SZCh	-	frequency normalization circuit
БПП	-	BPP	-	jamming transmitter
ОУ	-	OU	-	final amplifier
СЗ	-	SZ	-	delay circuit
ИШ	-	ISh	-	thyratron, noise diode
У-О	-	U-O	-	broad-band amplifier-limiter
ПУ	-	PU	-	threshold device
СФМИ	-	SFMI	-	modulating pulses

Designation for Chapter 3

ОУ – OU – final amplifier

Designation for Chapter 4

PCH — RSN — equisignal direction

Designation for Chapter 5

ВР	– VR	– time discriminator
СЗ	– SZ	– delay circuit
ГСН	– GSN	– strobe pulse generator
СЗЧ	– SZCh	– frequency memorization circuit
ОЧ	– OU	– final amplifier
Прм	– Prm	– receiver
ЛЗ	– LZ	– delay line
УУ	– UU	– control unit
Гем	– Gem	– oscillator
СМ	– SM	– mixer
УПЧ	– UPCh	– i-f amplifier
ФДЧ	– FDCh	– doppler filter
УФ	– UF	– narrow-band filter
РЛ	– RL	– reactance tube
ЧД	– ChD	– discriminator
ЛБВ	– LBV	– tw tube

Designations for Chapter 6

РПД	RPD	jamming
ЗУР	ZUR	guided antiaircraft missiles
ГСН	GSN	homing device
КРУ	KRU	command radio control link
СРП	SRP	computer
Прд	Prd	transmitter
Прм	Prm	receiver
ДШ	DSh	decoder
ИК	IK	actuating code
ХИП	KhIP	random pulse interference
СЗЧ	SZCh	frequency memorization circuit
УУ	UU	control unit
ОУ	OU	final amplifier

Designations for Chapter 7

ПП	-	PP	-	interference producer
$ПС_1$	-	PS_1	-	covered aircraft
АП	-	AP	-	duplexer
КГ	-	KG	-	coherent oscillator
МГ	-	MG	-	local oscillator
Прм	-	Prm	-	receiving device
ИКО	-	IKO	-	plan position indicator
ЛЗ	-	LZ	-	delay line

Designations for Chapter 8

ПУ - PU - preamplifier

ОУ - OU - final amplifier

Designations for Chapter 9

No explanation needed.

Designations for Chapter 10

ПРМ	PRM	receivers
АН	AN	analyzer
УЗО	UZO	unit for storage and processing information
ТУ	TU	telemetric device
ВЦ	VTs	input circuit
УВЧ	UVCh	high frequency amplifier
СМ	SM	mixer
УПЧ	UPCh	intermediate frequency amplifier
ВУ	VU	video amplifier
ЧР	ChR	frequency scanning
УНЧ	UNCh	low frequency amplifiers
АРУ	ARU	automatic gain control
ШУ	ShU	wideband amplifier
ФД	FD	phase detector
ЧД	ChD	frequency discriminator
РЛ	RL	reactance tube
ГП	GP	jamming generator
ЗГ	ZG	master oscillator
ИСЗ	ISZ	artificial earth satellites

CHAPTER 1

CRITERIA OF THE EFFECTIVENESS OF MEANS AND METHODS OF JAMMING

1.1. General Characteristic of the Criteria

In general jamming [RPD] by electronic means of antiaircraft defense [PVO] can be attained by the following ways:

– creation of radio interference (active and passive jamming false targets);

– change in electrical properties of the medium (ionization of space, the creation of absorbing and dispersing media);

– change in dispersing properties of the object (decrease in effective area of scattering, jamming camouflage).

Active jamming is created with the help of transmitters tuned to frequencies of suppressed electronic means, which are specially modulated in reference to specific objects of suppression. Interference signals can provide camouflage of the desired signal or its imitation. Accordingly camouflaging and simulating interferences are distinguished. Active interferences can also lead to a change in converting properties of appropriate electronic links.

Passive jamming is created at present by ejecting a large quantity of dipoles, effectively dispersing electromagnetic waves. The power of the signal reflected from the cloud of dipoles can considerably exceed the power of the signal from the aircraft.

False targets – radar traps – are aircraft (rockets) launched from aircraft or the ground having quite high effective areas of scattering. The latter is attained by application of special reemitters (passive or active).

The deliberate change in electrical properties of the medium can be achieved both by creating of artificially ionized regions and by inserting into the medium different absorbing and dispersing impurities (for example, smoke). The anomalies created cause in regions of their appearance disturbance of the usual conditions of the propagation of radio waves.

A change in dispersing properties of the object is attained by the application of various kinds of radar jamming coverages and reemitters of electromagnetic energy, as a result of which the detection of targets either becomes impossible or is hampered.

The means of jamming (radio interference, change in properties of the medium, decrease in radar contrast), as a result of their application, do not lead to material destructions and can only change the quantity of information circulating in the object of action. A change in the quantity of information passing through the electronic links leads to a change in the quantity of information in the whole control system, which in the end lowers the combat effectiveness of means of destruction of PVO serviced by the system. This essentially reflects the basic principle of RPD – a decrease by electronic methods of the combat effectiveness of means of destruction of the enemy by changing the quantity of information in his control system.

Consequently, to understand the general principles of RPD and determine the criteria of an estimate of their effectiveness, it is necessary to determine the depend .nce between parameters of RPD means and the degree of their influence on combat effectiveness of suppressed electronic systems [RES] of the enemy.

Two groups of criteria were determined – information and operational-tactical.

Information criteria permit estimating the quality of concrete interference signals and the quality of measures undertaken for applying information damage to the enemy.

Operational-tactical criteria are initial in the development of principles of armament by means of jamming. They permit estimating the quality of measures undertaken for the organization of jamming in combat and operation.

1.2. Information Criteria

Depending upon the form of interference signal and class of suppressed electronic means, different information criteria can take place.

It is convenient to estimate quality of masking interferences to radars operating in conditions of scanning with the help of the entropy of the interference signal. The expediency of such a criterion of the quality of the given form of interference signal can be shown in the following way.

Masking interference signals should exclude the possibility of detection of the desired signal with a probability exceeding the rated value under certain limiting conditions. An indispensable condition of the correct functioning of systems of information security known to the present time is the a priori knowledge of the desired signal. The degree of this knowledge can be diverse, but nonetheless certain a priori information on useful signals, and about laws of the distribution of particular forms of signals belonging to the given class should be always known. Otherwise, it is impossible to ensure the efficiency of the information system.

Ideal masking interference signals should create such conditions at which a posteriori after reception of the useful signal, the a priori uncertainty in the system of information security was preserved. The indicated property of masking interferences should take place over a prolonged time for different electronic means of a given class.

The given circumstances exclude the possibility of the application for these purposes of determined signals, inasmuch as they will be easily identified by the enemy and therefore cannot increase the indeterminancies in the system. Moreover, by comparatively simple technical procedures the determined interference signals can be eliminated, i.e., they possess low potential possibilities of camouflage (with the exception, perhaps, of such cases when their power is extraordinarily great). In other words, masking interference signals should contain an element of uncertainty. The greater the uncertainty of the interference signal at the assigned limitations, the less the potential possibilities for the enemy to eliminate it and with greater uncertainty the enemy must make a decision.

As is known [1, 2], the measure of uncertainty of a random variable or random process is entropy.

In the case of the discrete distribution of random the variable, described by the full finite probability scheme [2]:

$$\Pi = \begin{pmatrix} \Pi_1 \ldots \Pi_i \ldots \Pi_n \\ P_1 \ldots P_i \ldots P_n \end{pmatrix},$$

where Π_i – value of the random variable; P_i – probability of the fact that value Π_i will take place;

$$\sum_{i=1}^{n} P_i = 1,$$

entropy $H(\Pi)$ random variable Π is determined by formula

$$H(\Pi) = -\sum_{i=1}^{n} P_i \log P_i. \tag{1.1}$$

Other things being equal, among the camouflaging interference signals the best is that one whose entropy is greater.

If the random variable X is described by the continuous law of distribution with density p(x), then its entropy

$$H(X) = -\int_{-\infty}^{+\infty} p(x) \log p(x)\, dx. \qquad (1.2)$$

Accordingly, for the random variable characterized by multidimensional density of distribution $p(x_1, \ldots, x_n)$:

$$H(X) = -\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(x_1, \ldots, x_n) \log p(x_1, \ldots, x_n)\, dx_1 \ldots dx_n. \qquad (1.3)$$

The introduction of entropy as a characteristic of the quality of masking interference signals permits estimating potential possibilities of interferences regardless of the concrete methods of their treatment in suppressed devices. A direct characteristic of quality of the use of available power for the creation of masking interferences (creation of uncertainty) is the entropy power of the interference signal. This idea is discussed in detail in the second chapter. Let us note that the application of entropy of interference signals permits estimating to a certain degree their potential interference possibilities.

The quality of the simulating interferences and false targets can be estimated in the following way. The true target is characterized by a certain large number of independent parameters. These parameters can be accepted as coordinates of the multidimensional space. The signal of a corresponding target can be represented in the form of a vector in this space. Thus the true target can be brought to conformity as the vector of criteria $Ц(\alpha_1, \ldots, \alpha_n)$, where $\alpha_1, \ldots, \alpha_n$ – independent parameters quantitatively characterizing the indicated criteria.

If $\alpha_1, \ldots, \alpha_n$ would be completely determined values, the problem of the creation of false marks would lead to a corresponding reproduction with the help of the interference device of components of the vector of criteria.

For real targets each of criteria α_k can constitute either a random variable, distributed over great number of true targets, or

a random time function for each given target.

In other words, on a certain part or on all independent parameters (components of the vector of criteria) there is assigned the probability distribution

$$a_k = \begin{pmatrix} a_k^1 \dots a_k^N \\ P_1 \dots P_N \end{pmatrix},$$

$$\sum_{j=1}^{N} P_j = 1.$$

Here P_j – probability of the k-th criterion takes the value a_k^j.

The indicated circumstance makes necessary the introduction of uncertainty into the vector of criteria describing the false mark (target).

In other words, it is necessary to assign, in the appropriate way, the probability distribution to corresponding components of the vector of criteria of the false target.

Let us assume that great numbers of true targets Ц/И of the given class and false targets of the same class Ц/Л each contain m elements $(Ц_1/И, \dots, Ц_m/И)$, $(Ц_1/Л, \dots, Ц_m/Л)$, and let us assume that there is known a priori the probability distribution on great numbers of Ц/И and Ц/Л, which form the full probabilistic scheme assigned on the assumption that the target is represented by the k-th criterion:

$$Ц/И = \begin{pmatrix} Ц_1/И \dots Ц_m/И \\ P_{j1}(Ц/И) \dots P_{jm}(Ц/И) \end{pmatrix},$$

$$\sum_{i=1}^{m} P_{ji} = 1.$$

An analogous probabilistic scheme can be recorded for a great number of false targets. Each of the indicated probabilistic schemes brings into conformity the entropy determined under the condition that both the true and false targets are represented by k-th criteria, i.e., that for both true and false targets the full probabilistic

scheme α_k. The quality of reproduction $I(Ц_k/Л)$ of the true target with the help of the false target, represented by the k-th criterion, can be estimated by the difference in conditional entropies - conditional entropy of the false target $H(Ц_k/Л)$, represented by the k-th criterion, and the conditional entropy of the true target, also represented by the k-th criterion:

$$I(Ц_k/Л) = H(Ц_k/Л) - H(Ц_k/И). \tag{1.4}$$

Here

$$H(Ц_k/Л) = -\sum_{j=1}^{N} P_j(\alpha_k/Л) \sum_{i=1}^{m} P_{ji}(Ц_i/Л) \log P_{ji}(Ц_i/Л); \tag{1.5}$$

$$H(Ц_k/И) = -\sum_{j=1}^{N} P_j(\alpha_k/И) \sum_{i=1}^{m} P_{ji}(Ц_i/И) \log P_{ji}(Ц_i/И); \tag{1.6}$$

$P_j(\alpha_k/Л)$ — conditional probability to the k-th criterion of the false target takes the value $\alpha_k^j (j = 1, 2, \ldots, N;\ k = 1, 2, \ldots, m)$; $P_j(\alpha_k/И)$ — conditional probability of the k-th criterion of the true target takes the value α_k^j; $P_{ji}(Ц_i/Л)$ - conditional probability of the presence of the i-th false target with the corresponding criterion taking the value $\alpha_k^j/Л$; $P_{ji}(Ц_i/И)$ — conditional probability of the presence of the ith true target with the given criterion taking the value $\alpha_k^j/И$.

Frequently it is convenient to present the quality of reproduction with the help of the difference in conditional entropies obtained by averaging not with respect to the probability distribution into regions of the determination of one the criteria, but by averaging over a great number of targets . Then in virtue of the symmetry, we will obtain

$$I(Ц_k/Л) = H(Л/Ц_k) - H(И/Ц_k), \tag{1.7}$$

where

$$H(Л/Ц_k) = -\sum_{i=1}^{m} P_i(Ц_i/Л) \sum_{j=1}^{N} P_{ji}(\alpha_k/Л) \log P_{ji}(\alpha_k/Л); \tag{1.8}$$

$$H(И/Ц_k) = -\sum_{i=1}^{m} P_i(Ц_i/И)\sum_{j=1}^{N} P_{ji}(a_k/И)\log P_{ji}(a_k/И). \quad (1.9)$$

The equality of conditional entropies

$$H(Л/Ц_k) = H(И/Ц_k)$$

is a necessary and sufficient condition so that the false target would completely present the true one by the k-th criterion [3, 4].

A convenient characteristic of the discernibility of two hypotheses and also of the quality of imitation of the true target with the help of a false one is the so-called divergence of Kul'bak [Translator's note: Western spelling could be Koolback] [3, 5].

It permits qualitatively estimating the quality of description of the true target represented by the vector of criteria $Ц_И(a_И^1, \ldots, a_И^N)$ and full probabilistic scheme for discrete values of each of the n criteria

$$a_И^k = \begin{pmatrix} a_k^1 \ldots a_k^j & \ldots a_k^N \\ P_1(a_k^1/И) \ldots P(a_k^j/И) \ldots P(a_k^N/И) \end{pmatrix},$$

with the help of the vector of criteria of the false target $Ц_Л(a_Л^1, \ldots, a_Л^N)$, the components of which are described by the corresponding full probabilistic schemes

$$a_Л^k = \begin{pmatrix} a_k^1 & \ldots & a_k^j & \ldots a_k^N \\ P(a_k^1/Л) \ldots P(a_k^j/Л) \ldots P(a_k^N/Л) \end{pmatrix}.$$

In accordance with the accepted designations the Kul'bak divergence (distance between hypotheses) is recorded in the following way:

$$\mathrm{Div}\, a_k = \sum_{j=1}^{N} [P_j(a_k/И) - P_j(a_k/Л)] \log \frac{P_j(a_k/И)}{P_j(a_k/Л)}. \quad (1.10)$$

For indisceribble targets or identical descriptions, when $P_j(\alpha_k/И)=P_j(\alpha_k/Л)$, the Kul'bak divergence turns into zero.

The Kul'bak divergence will be compared favorably with the entropy measure by the fact that it permits obtaining simpler calculation formulas in the case of normal laws of distribution. The Kul'bak divergence is completely determined by a posteriori distributions.

The aforementioned formula for the Kul'bak divergence (1.10) can be obtained by the following reasonings. Let us assume that there is assigned the a priori probability distribution of discrete values of one of the components of the vector of criteria $P(\alpha_k)$ of the true target, and let us assume that the a posteriori probability distribution of discrete values of this criterion for the following two alternative hypotheses is also known: the criterion corresponds to the true target $P(\alpha_k/И)$, and the criterion corresponds to the false target (mark) $P(\alpha_k/Л)$.

Then the quality of information $I(\alpha_k^i, \alpha_k^i/И)$ contained in value $\alpha_k^i/И$ on value α_k^i will be defined as the logarithm of the ratio of the a posteriori probability to the a priori probability. The base of the logarithm is selected different, depending upon what units of measurement of information are preferred to be used [6]:

$$I(\alpha_k^i; \alpha_k^i/И) = \log \frac{P(\alpha_k^i/И)}{P(\alpha_k^i)}. \tag{1.11}$$

There will similarly be determined the quantity of information in the random variable $\alpha_k^i/Л$, which refers to the false target, on the random variable α_k^i:

$$I(\alpha_k^i; \alpha_k^i/Л) = \log \frac{P(\alpha_k^i/Л)}{P(\alpha_k^i)}. \tag{1.12}$$

The difference of (1.11) and (1.12) will determine the measure of conformity of the false target by the ture, if they are both represented by the random value of one of the components of the vector of criteria:

$$I(\alpha_k^i;\alpha_k^i/И)-I(\alpha_k^i;\alpha_k^i/Л)=\log\frac{P(\alpha_k^i/И)}{P(\alpha_k^i/Л)}. \tag{1.13}$$

More accurately this difference determines the quantity of information included in the random variable α_k^i, in favor of the hypothesis about the fact that the given target is the true one as compared to the alternative hypothesis (false target). If we average the obtained value of difference (1.13) over all possible values α_k^i separately for the probabilistic scheme of the k-th criterion of the true target and the same criterion of the false target, then accordingly we will obtain the average quantity of information in favor of the hypothesis about the fact that the given criterion represents the true target. Therefore,

$$\sum_{i=1}^{N} P(\alpha_k^i/И)\log\frac{P(\alpha_k^i/И)}{P(\alpha_k^i/Л)} \tag{1.14}$$

is the average quantity of information included in the random variable α_k^i in favor of the fact that the identified target is true at the time when this corresponds to reality. Similarly

$$\sum_{i=1}^{N} P(\alpha_k^i/Л)\log\frac{P(\alpha_k^i/И)}{P(\alpha_k^i/Л)} \tag{1.14a}$$

is the average quantity of information, included in the random variable α_k^i in favor of the fact that the identified target is true, whereas in reality it is false.

The difference between (1.14) and (1.14a) is equal to Div α_k [see (1.10)]. It determines the value of divergence between alternative hypotheses relative to the situation described by the k-th component of the vector of criteria.

In virtue of the independence of a priori probability distributions, the divergence of alternative hypotheses is a very convenient criterion of quality not only of simulating but also masking interferences. In order to apply this criterion of quality for masking

interferences, it is necessary to know the conditional a posteriori probability distribution for the realization representing only the interference signal $p(x_1, \ldots, x_n/\Pi)$ and realization containing a mixture of interference and desired signals $p(x_1, \ldots, x_n/C)$. If multidimensional densities of distribution are differentiable and are determined on the whole real axis, then

$$\mathrm{Div}\,\Pi/C = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} [p(x_1, \ldots, x_n/C) - \\ - p(x_1, \ldots, x_n/\Pi)] \log \frac{p(x_1, \ldots, x_n/C)}{p(x_1, \ldots, x_n/\Pi)} dx_1 \ldots dx_n. \quad (1.15)$$

The convenience of the aforementioned information criteria of the quality of interference signals consists, first of all, in the fact that for the developer of interference means in practice there is always the necessary information for carrying out concrete calculations according to these criteria.

A special advantage of the examined information criteria, as was already noted earlier, consists in the fact that they permit estimating the quality of interference signals without a tying to the concrete suppressed devices and principles of the acceptance of the solution by the enemy under conditions of interferences. In order to apply these criteria to estimating the quality of simulating interference signals and false targets, it is necessary to know the a posteriori estimators of the latter.

1.3. Power Characteristics of Interference Signals

An important power characteristic of interference signals is the coefficient of suppression. Sometimes the coefficient of suppression is called the power criterion of the quality of interference signals. It is, however, expedient to examine the coefficient of suppression not as an independent criterion but as a power characteristic of the assigned interference signal and suppressed means.

By the coefficient of suppression is understood the minimum necessary ratio of energy of the given interference signal to the

energy of the useful signal at the input of the receiving device of the suppresses REM in the passband of its linear part at which the assigned information loss takes place[1].

The information loss generated by the action of interferences appears in the camouflage, simulation and formation of errors and breaks in the entering of information and others. The character of the information loss depends on the form of the interference signal and suppressed means.

The assigned (acceptable in a certain meaning) information loss is determined preliminarily with the help of operational-tactical criteria.

In many cases the useful signals can be examined as pulses of rectangular form (especially in radar). In these conditions it is convenient to express the coefficient of suppression in terms of the ratio of powers of the interference and useful signals at the input of the receiving device:

$$k_{п}=\left(\frac{P_{п}}{P_{c}}\right)_{вх\ мин}. \qquad (1.16)$$

Here $P_{П}$ – power of the interference signal; P_{c} – pulse power of the useful signal.

For example, for white Gaussian noise, $P_{П}$ is equal to the product of its spectral density G on the passband of the linear part of the receiver $\Delta f_{пр}$, i.e.,

$$P_{п}=G\Delta f_{пр}.$$

In the case of pulse interferences $P_{П}$ is the power of the interference pulse if it has rectangular form.

[1]The therm "coefficient of suppression" was proposed by B. D. Sergiyevskiy.

If the useful signal is a continuous oscillation of constant amplitude, which takes place, for example, with frequency or phase modulation, then by P_c instantaneous power of the signal is understood.

Subsequently we will use determination (1.16). The concept of the coefficient of suppression in the form of the minimum necessary ratio of powers of interference and useful signals can be used for signals of arbitrary form if by P_Π and P_c the mean values of powers during the time equal to the average duration of the signals are understood. Numerical values of the coefficient of suppression can be found only for the assigned interference signal and assigned suppressed device.

Thus, the power criterion in contrast to the information criterion requires knowledge of concrete characteristics of the suppressed systems.

If the system is well-known, it can be suppressed with lower power consumptions by applying appropriate interference signals, not certainly optimum in the information criterion.

When probabilistic characteristics of interference and useful signals are known, and characteristics of the conversion of the signal and interference in the electronic device are known, then one can determine the minimum necessary power relationships with the help of the theory of statistical solutions. In particular, for masking interferences the coefficient of suppression is found in two stages. Initially, with respect to information criteria, the best quality of the interference signal is provided. After that for the signal optimum with respect to the information criterion is the coefficient of suppression by it of the given electronic device. The obtained numerical value of the coefficient will be approximate, and the degree of approximation for different criteria of acceptance of the solution will be different.

As is known [7, 8], the choice between two alternative hypotheses (interference or signal + interference) on the basic of consideration of the given selection (realization) of random voltage (current) obtained on the interval of observation (0, T), which is the sum of the useful and interference signal, can be performed with the help of a number of criteria (Bayes, minimax, Neumann-Pearson, Kotelnikov-Zigert and Wald). In all criteria the solution is taken according

to the magnitude of the likelihood ratio

$$\Lambda(v_1, \ldots, v_n) = \frac{p_1(v_1, \ldots, v_n)}{p_0(v_1, \ldots, v_n)}, \tag{1.17}$$

where $p_1(v_1, \ldots, v_n)$ and $P_0(v_1, \ldots, v_n)$ – multidimensional density of the distribution of voltage (current) taking place accordingly in the case of the additive mixture of the signal and noises and only one noise.

In making the choice between the two alternative hypotheses with respect to the given selection, the one making the decision can allow an error of two kinds.

Error of the first kind (false alarm). It is assumed that the second hypothesis (interference + the signal takes place) is correct when the first hypothesis (only interference takes place) is correct.

Error of the second kind (passing of the target). It is assumed that the first hypothesis is correct, whereas the second hypothesis takes place.

To take the solution in this case means to determine the borders of region R_0 of values of parameters of the selection (realization) $v_1, \ldots, v_n$, which correspond to the first hypothesis, and borders of region R_1 of values of these parameters corresponding to the second hypothesis.

The probability of error of the first kind (probability of false alarm) Q_0 will be determined by means of integration over region R_1 of the density of distribution $p_0(v_1, \ldots, v_n)$:

$$Q_0 = \int_{R_1} p_0(v_1, \ldots, v_n)\,dv_1 \ldots dv_n. \tag{1.18}$$

The probability of error of the second kind (probability of passing Q_1 will be determined by means of integration over region R_0 of the density of distribution $p_1(v_1, \ldots, v_n)$

$$Q_1 = \int_{R_0} p_1(v_1, \ldots, v_n)\,dv_1 \ldots dv_n. \tag{1.19}$$

Depending upon the applied criterion the likelihood ratio is selected in order to ensure probabilities of errors of the first and

second kind acceptable by certain considerations. In order optimally to make a choice between two alternative hypotheses ("only interference"-"interference + signal") with the help of the Bayes criterion, the one making the decisions should know the average risk

$$\bar{C}=\xi C_0 Q_0(R_1)+(1-\xi) C_1 Q_1(R_0), \tag{1.20}$$

where ξ – a priori probability of the correctness of the first hypothesis (only interference); $(1 - \xi)$ – a priori probability of the correctness of the alternative hypothesis (interference + signal); C_0 – cost of error of the first kind (false alarm) expressed in arbitrary units of measurement (it can be expressed in rubles); C_1 – cost of error of the second kind (passing of the target), expressed in the same units of measurement as those of C_0.

The observer, using the Bayes criterion, selects the border between regions R_0 and R_1 in such a way in order to ensure the minimum of average risk $\bar{C}$. The likelihood ratio corresponding to this condition is called threshold and is denoted by Λ_0:

$$\Lambda_0(v_1, \ldots, v_n)=\frac{p(v_1, \ldots, v_n)}{p_0(v_1, \ldots, v_n)}. \tag{1.21}$$

The magnitude of the threshold value of the likelihood ratio in the case of one-dimensional random variables $p_0(v)$ and $p_1(v)$ is found by means of differentiation of the expression for the average risk with respect to $R_0 = v_0$. The formula for the average risk with the choice between two alternative hypotheses on the basis of analysis of two random variables which are represented by one-dimensional laws of distribution $p_0(v)$ and $p_1(v)$, has the form

$$\bar{C}=\xi C_0\int_{v_0}^{\infty} p_0(v)\,dv+(1-\xi) C_1 \int_{-\infty}^{v_0} p_1(v)\,dv. \tag{1.22}$$

Here $p_0(v)$ – density of distribution of the random variable v if the first hypothesis is correct; $p_1(v)$ – density of distribution of the random variable v if the second hypothesis is correct.

Differentiating $\bar{C}$ with respect to v_0 and equating the derivative to zero, we will define the conditions at which minimum $\bar{C}$ is ensured. Existence of the minimum is easily proven by direct analysis of the formula for the average risk (1.22).

It turns out that $\bar{C} = \bar{C}_{\text{мин}}$, if

$$\frac{p_1(v_0)}{p_0(v_0)} = \frac{\xi C_0}{(1-\xi)C_1} = \Lambda_0(v_0). \tag{1.23}$$

The quantity $\Lambda_0(v_0)$ in the given example is the threshold value of the likelihood ratio

$$\Lambda = \frac{p_1(v)}{p_0(v)}.$$

The observer, in using the Bayes criterion, functions in the following way. According to the accepted realization likelihood ratio $\Lambda(v_1, \ldots, v_n)$ is determined, which is compared with the threshold value $\Lambda_0(v_1, \ldots, v_n)$. If

$$\Lambda(v_1, \ldots, v_n) < \Lambda_0(v_1, \ldots, v_n), \tag{1.24}$$

then the first hypothesis is taken, otherwise, the second hypothesis is taken.

The threshold value of the likelihood ratio can be brought into conformity by the ratio of energy of the useful signal to the energy of the interference signal. This ratio in radar is called the coefficient of discrimination.

The one creating interferences or developing the interference equipment must be interested in such ratios of energy of interference and energy of the signal (if the question is masking interferences) at which inequality (1.24) takes place. The threshold value of the likelihood ratio can also be brought into conformity by a certain value of the ratio of energies of the interference and useful signals, which should be examined as the minimum necessary. This minimum

necessary ratio, with certain additionally imposed limitations, which was discussed earlier (account of the passband of the linear part of the receiver), determines the coefficient of suppression. It is easy to see that the coefficient of suppression for masking interferences is reciprocal of the coefficient of discrimination.

Numerical values of the coefficient of suppression are determined, as a rule, approximately, inasmuch as those valves creating interferences are the unknown accurate values of coefficients C_0 and C_1 also a priori probabilities ξ and $(1 - \xi)$ on which the suppressed side is oriented.

The a priori probability ξ can be unknown and suppressed to the side. Then the one who makes the decision must make different assumptions about the enemy. One of the variants of such reasonings leads to the so-called minimax criterion the essence of which can be clarified in the following way.

Let us assume that the one making the decision on the suppressed radar does not know the a priori probability ξ and arbitrarily selects $\xi = \xi_1$. In this case the value of the average risk for the one making the decision will be determined by formula (1.20), recorded for one-dimensional distribution, in which instead of $Q_0(v_0)$ and $Q_1(v_0)$ it is necessary to substitute their values corresponding to the accepted value of the a priori probability $\xi = \xi_1$.

The indicated substitution is conditioned by the fact that quantity v_0, minimizing the average risk $\bar{C}(v_0)$, in virtue (1.23) is determined by value $\xi = \xi_1$:

$$\bar{C}(\xi_1) = \xi_1 C_0 Q_0 [v_0(\xi_1)] + (1 - \xi_1) C_1 Q_1 [v_0(\xi_1)]. \tag{1.25}$$

In the case when the value of the a priori probability ξ is not equal to ξ_1, the value of the average risk can appear both considerably more and less than $\bar{C}(\xi_1)$.

Figure 1.1 shows the approximate dependence of the average risk $\bar{C}(\xi)$ on the a priori probability ξ. Every point of this curve is

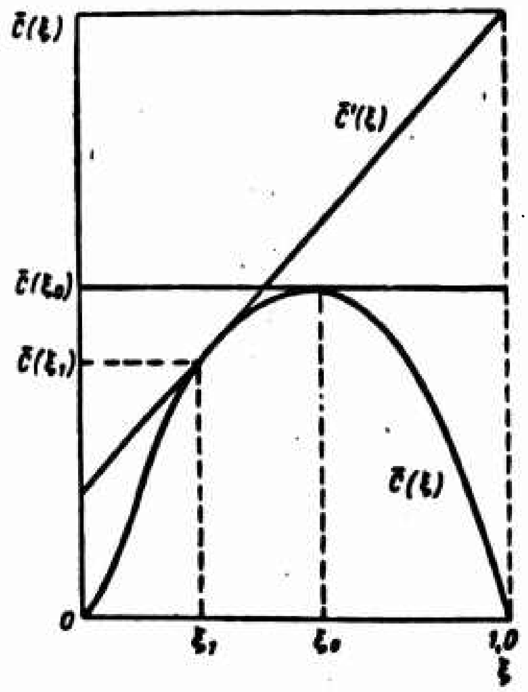

Fig. 1.1. Dependence of average rick ($\bar{C}$) for the one making the decision on the a priori probability ξ in the case of the Bayes and minimax criteria.

the minimum average risk corresponding to the given value of the a priori probability ξ. The selected value ξ_1 on the axis of the ordinates corresponds to the minimum value of the average risk $\bar{C}(\xi_1)$. If the value of the a priori probability ξ is not equal to ξ_1, according to which there is carried out the minimization of average risk $\bar{C}(\xi_1)$, then the corresponding average risk $\bar{C}'(\xi)$ will be determined by ordinates of points of a straight line tangent to curve $\bar{C}(\xi)$ at point ξ_1, i.e., $\bar{C}'(\xi_1) = \bar{C}(\xi_1)$. The equation of this straight line is obtained from formula (1.25) if in it instead of factors ξ_1 and $(1 - \xi_1)$ there is respectively substituted ξ and $(1 - \xi)$:

$$\bar{C}'(\xi) = \xi C_0 Q_0 |v_0(\xi_1)| + (1 - \xi) C_1 Q_1 |v_0(\xi_1)|. \tag{1.26}$$

In order not to allow losses greater than $\bar{C}(\xi_0)$ corresponding to the maximum of curve $\bar{C}(\xi)$, the one making the decision should be oriented on the value of the a priori probability $\xi = \xi_0$. In this case the straight line, which determines values of the average risk

$\bar{C}'(\xi_0)$ for values $\xi \neq \xi_0$, will be parallel to the axis of the abscissas, i.e., the average risk at any values of ξ will not exceed $\bar{C}(\xi_0)$. Inasmuch as $\bar{C}(\xi_0)$ is the minimum average risk corresponding a priori to probability $\xi = \xi_0$ and, furthermore, the maximum among all minimized a priori probabilities of average risk, it was agreed to call it the minimax average risk. Later this term will again be encountered in connection with the consideration of certain theoretical game problems.

In order to find the threshold value of the liklihood ratio and boundary value $v = v_0$ corresponding to it for the minimax criterion, it is necessary to differentiate the expression for average risk (1.22) with respect to ξ and equate the derivative to zero. The obtained transcendental equation

$$C_0 Q_0 (v_0) = C_1 Q_1 (v_0) \tag{1.27}$$

permits finding the sought value v_0 corresponding to the maximum of the minimized average risk and determining the threshold likelihood ratio

$$\Lambda_0 (v_0) = \frac{p_1 (v_0)}{p_0 (v_0)}.$$

It is obvious that, being oriented on tha a priori probability $\xi = \xi_0$, the one creating interferences assumes work in conditions most profitable for himself . The coefficient of suppression corresponding to ξ_0 can appear smaller than for any other values of ξ.

Application of the minimax criterion does not remove the uncertainty in calculations of the coefficient of suppression, inasmuch as the interferences, as a rule, are unknown values of costs C_0 and C_1 on which the suppressed side is oriented.

The Kotelnikov-Zigert criterion ("ideal observer") assumes the equality of costs of errors of the first and second kind ($C_0 = C_1$). In this case minimization of the average risk is equivalent to minimization of the full probability of acceptance of an erroneous

decision P_{om}. The "ideal observer" selects the border between regions R_0 and R_1 in such a way in order to minimize the average probability of an erroneous decision

$$P_{om}=\xi Q_0+(1-\xi)Q_1. \tag{1.28}$$

The Kotelnikov-Zigert criterion is applied in systems of radio communications.

Just as in the case of the Bayes criterion, the one organizing the interferences, in calculating the coefficient of suppression by the criterion of the "ideal observer," can allow an error in virtue of the inaccurate knowledge of the a priori probability ξ, on which the one being suppressed is oriented.

If in radio communications it is permissible to estimate equally errors of the first and second kind, then in radar the false alarm and passing of a target are events of fundamentally different significance. Furthermore, in radar there are encountered difficulties with determination and often with simple interpretation of a priori probabilities. In virtue of the indicated circumstances in radar the basic criterion for determining of the threshold likelihood ratio and coefficient of discrimination corresponding to it is the Neumann-Pearson criterion.

The Neumann-Pearson criterion requires to select so the boundary between regions R_0 and R_1 in order to provide a minimum of probability of the passing of a signal at the assigned probability of the false alarm. Mathematically the problem is reduced to the determination of the conditional extremum of the function of many variables $Q_1(v_1, \ldots, v_n)$ with a limitation of the form

$$Q_0(v_1, \ldots, v_n)=\text{const}.$$

The conditional extremum is usually found by the method of indefinite Lagrange multipliers. In the examined case of one limiting condition the initial linear combination has the form

$$Q_1(R_0)+\lambda Q_0(R_1), \tag{1.29}$$

here λ — indefinite Lagrange multiplier.

It is necessary to find the border between regions R_0 and R_1 in such a way in order that the minimum of the linear combination (1.29) is ensured.

The linear combination (1.29) coincides with the expression for the average risk (1.20) if one were to assume $C_0 = 2\lambda$, $C_1 = 2$, and $\xi = 1/2$. Earlier it was established that the minimum of average risk takes place if the border between regions R_0 and R_1 is selected so that equality (1.23) for the liklihood ratio takes place.

If equivalence of expressions (1.29) and (1.20) takes place under the conditions indicated above, it is possible simply to affirm that the minimum of the linear combination (1.29) will take place when the border between regions R_0 and R_1 will ensure the fulfillment of the following equality:

$$\frac{p_1(v_1, \ldots, v_n)}{p_0(v_1, \ldots, v_n)} = \Lambda_0(v_1, \ldots, v_n) = \lambda. \tag{1.30}$$

Quantity $\lambda = \Lambda_0(v_1, \ldots, v_n)$ is selected in such a way in order to ensure the rated value of probability of the false alarm

$$Q_0 = \int_{\lambda_0}^{\infty} p_0(\Lambda)\, d\Lambda. \tag{1.31}$$

The last formula directly follows from equation (1.18) if one were to consider that the likelihood ratio $\Lambda(v_1, \ldots, v_n)$ is the random variable, which varies from realization to realization under the condition of correctness of the same hypothesis, namely, the hypothesis cn the fact that only interferences take place.

The density of distribution $p_0(\Lambda)$ can be determined if one were to consider that Λ is function $p_0(v_1, \ldots, v_n)$, namely:

$$\Lambda(v_1, \ldots, v_n) = \frac{p_1(v_1, \ldots, v_n)}{p_0(v_1, \ldots, v_n)}.$$

It follows from this that with the correctness of the first hypothesis

$$p_0(\Lambda)d\Lambda = p_0(v_1, \ldots, v_n)dv_1 \ldots dv_n.$$

Further, with the help of equation (1.18), taking into account (1.30), we obtain formula (1.31).

The Neumann-Pearson observer functions in the following way. According to a given selection (realization) likelihood ratio Λ is determined. If Λ is greater than Λ_0, determined by the assigned probability of the false alarm Q_0 with the help of formula (1.31), then the second hypothesis is taken; otherwise, the first hypothesis is considered correct.

The one creating interferences, in calculating the coefficient of suppression in accordance with the Neuman-Pearson criterion, can allow error owing to the inaccurate knowledge of the value of the probability of a false alarm accepted on the suppressed side. In practical calculations of the coefficient of suppression one should never orient on the easiest conditions. The value of the coefficient of suppression should be selected with such calculation in order to provide suppression of the corresponding electronic means in conditions most unfavorable for the one creating interferences if the probability of the existence of such conditions is quite great (less than 0.5).

Naturally, the question arises as to the possibility of establishing communication between the information criteria of the quality of masking interferences and criteria of the theory of decisions, which permit calculating the value of coefficients of suppression for a given form of the interference signal.

Information criteria of the quality of interference signals and power criteria are different in their nature, and there is no direct functional dependence between them. However, the knowledge of

certain information criteria of the quality of interference signals imposes quite definite limitations on the possible range of values of corresponding criteria of the statistical decision theory.

Thus, for example, with the assigned average probability of error $P_{\text{ош}}$, determined by the Kotelnikov-Zigert criterion maximum entropy is determined by the dependence [3, 9]

$$H_{\text{макс}}(P_{\text{ош}}) = -P_{\text{ош}} \log P_{\text{ош}} - (1 - P_{\text{ош}}) \log (1 - P_{\text{ош}}) + \\ + P_{\text{ош}} \log (n - 1). \tag{1.32}$$

where n — number of elements of the corresponding probabilistic scheme determining $H(P_{\text{ош}})$.

The examination conducted indicates the necessity of the application in jamming of both information criteria and criteria of the theory of decisions, in contrast to radar where preference is given to criteria of the decision theory.

Criteria of the decision theory in radar are more convenient, especially when the question is about the detection of the signal against a background of own noise, the estimators of which are known, the average risk or the permissible probability of false alarms can somehow be determined.

However, in the case of deliberate interferences with unknown a priori probability distribution, the application of these criteria is difficult. Even with electronic computers the likelihood ratio with a priori unknown distribution of interference is far from always determined. It is necessary to consider also that under conditions of deliberate interferences the decision on the presence of a target on the radar screen, as a rule, will be made by the operator.

At present it is impossible to affirm simply that the operator estimates precisely the value of the likelihood ratio. It is more justified to consider that the operator is basically guided by a priori knowledge about the useful signal and interferences.

The given considerations permit considering the information criteria acceptable characteristics of quality of the reception of signals in radar occurring under the influence of organized interferences. Even more justified is the application of these criteria for an estimate of the quality of interference signals.

At the same time one should again stress the necessity to have power characteristics of interference signals optimized by the information criterion, which allow producing appropriate calculations of interference means.

1.4. Peculiarities of Criteria of the Quality of Interference Signals Intended for the Suppression Automatic Control Systems

At present from the point of view of the one creating interferences, all automatic control systems can be divided into four basic groups:

- deterministic;

- statistically defined;

- systems with adaptation;

- game or minimax.

We will refer to the deterministic as such control systems whose algorithm of functioning is known either a priori or can be determined in the process of the creation of interferences, and, furthermore a great number of permissible interference signals is known[1]. It, just as the algorithm of functioning can be determined either a priori with the help of the equipment of electronic reconnaissance.

[1]To the permissible signals pertain such interference signals which can have on the controlled object the same influence as the useful controlling signals.

In conformity with the determination of the one creating the interference the algorithm of functioning of the deterministic control system is considered well-known.

Usually the algorithm of functioning of the deterministic system constitutes the totality of ordinary differential equations describing the process of travel of the controlled object. In normal form the indicated system of differential equations has the following form:

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(x_1, \ldots, x_n;\ u_1, \ldots, u_r), \\ &\cdots\cdots\cdots\cdots\cdots \\ \frac{dx_n}{dt} &= f_n(x_1, \ldots, x_n;\ u_1, \ldots, u_r). \end{aligned} \tag{1.33}$$

Here $x_1, \ldots, x_n$ – phase coordinates of the controlled object in n-th dimensional phase space (they are unknown time functions);

$u_1, \ldots, u_r$ – position of regulating units ensuring the possibility of control of the object.

On coordinates $u_1, \ldots, u_r$ limitations are imposed. Each of the coordinates $u_j (j = 1, \ldots, r)$ should belong to a great number of permissible controls $U_R (u_j \in U_R)$, i.e., it cannot be more or less the certain extremal limits $u_{j\,\text{мин}} < u_j < u_{j\,\text{макс}}$. Functions $f_i(x_1, \ldots, x_n;\ u_1, \ldots, u_r)$ are determined on all possible (for the given system) values of phase coordinates $x_i (i = 1, 2, \ldots, n)$ and values $u_j (j = 1, 2, \ldots, r)$ belonging to a great number of permissible controls.

In the theory of optimum control [10, 11, 12] the quality of control is estimated with the help of integral functionals of the following form:

$$Q = \int_{t_0}^{T} G(x_1, \ldots, x_n;\ u_1, \ldots, u_r)\, dt. \tag{1.34}$$

Here $G(x_1, \ldots, x_n;\ u_1, \ldots u_r)$ is a certain assigned function.

For every permissible control $u_1(t), \ldots, u_r(t)$, assigned on the time interval $t_0 < t < T$, the movement of controlled process is uniquely

determined, and the integral functional corresponding to this movement of the process takes a certain value. In other words, every permissible control during the time $t_0 < t < T$ transfers the system from the initial phase state $x_i(t_0)(i = 1, 2, \ldots, n)$ into the final state $x_i(T)$, and by this transition of the system from one point of the phase space into the other the numerical value of the integral functional Q can be brought into a one-to-one conformity.

Control $u_j(t)$, where $j = 1, 2, \ldots, r$, is called optimum, if it is permissible and the outer limit of the functional Q corresponds to it.

Usually in the theory of optimum control rush we try to provide the minimum of the integral functional. For example, it is possible to require that the optimum control $u_j(t)$ provides a transfer of the system from one phase state into the other in minimum time. The integrand and integral functional will be converted accordingly:

$$G(x_1, \ldots, x_n; u_1, \ldots, u_r) = 1,$$
$$Q = \int_{t_0}^{T} dt = T - t_0. \tag{1.35}$$

In this case the control which is optimum in high-speed operation is indicated.

Interference signals to deterministic systems of automatic control directly act on controls of the object and can be examined as permissible controls. Usually the power of the interference signal occurs considerably more than the power of the useful signal, and with action of the latter it can practically be not considered.

Permissible controls generated by interference signals will be called interference controls.

A great number of interference controls is determined by the form of the interference signals, parameters of interference equipment and the suppressed automatic system. In other words, it can be limited either by permissible limits of the deflection of controls

of the automatic system or by ensured possibilities of the interference equipment.

The criterion of the quality of interference control can be the same functional (1.34); however, the target of the interference control is different from the target of the usual (useful) control. If, for example, the usual control is to minimize the time of transition of the system from a certain point of the phase space into the origin of the coordinates (which, usually, corresponds to the minimum of the error of the system), then the interference control, conversely, tries in minimum time to transfer the system from the origin of the coordinates into such a point of the phase space which corresponds to the sufficient value of the error with respect to the operational-tactical criterion.

By coefficient of suppression of systems of automatic control we imply the minimum necessary ratio of energies of the interference and useful signals at the input of the suppressed control system within limits of the passband of the linear part of the receiver at which the possibility of transferring the system from the origin of the coordinates into the assigned region of the phase space is provided.

In practice it is far from always possible to form the optimum interference control. Often we are limited to interference controls providing in principle the transfer of the suppressed system of automatic control from the origin of the coordinates into a certain assigned region of the phase space. Such interference controls are called sufficient controls. In subsequent chapters of the book sufficient interference controls will basically be studied.

Statistically defined systems differ from deterministic systems by the fact that parameters of suppressed systems are not accurately known to the one creating interferences, although the principle of the functioning of them is known and laws of the probability distribution of possible values of parameters of these systems are known. An example of a similar kind of system can be a circuit of homing

guidance with parameters of separate sections inaccurately known to the one creating the interference.

Differential equations, describing the process of functioning of the statistically defined system, are similar to the system of equaitons (1.33) with the only difference being that variables x_i, f_i, and u_j entering into them will be random. The law of distribution $f_i (i = 1, 2, \ldots, n)$ is a priori known. The criterion of the quality of interference signals in this case can be the integral functional of the type (1.34), but the only determining mathematical expectation is Q.

Usually for statistically defined systems interference controls are basically sufficient.

The functioning of automatic control systems with adaptations can also be described with the help of the system of differential equations of the type (1.33) but only during a time until the interferences act. As soon as on the system with adaptations there starts to act the assigned interference, the system of equations of the type (1.33) is converted into the form corresponding to the given interference signal. Every form of interference signal, from a certain class of signals, is brought into conformity with the system of differential equations describing the functioning of the automatic device under conditions of the influence of the given form of interferences.

The criterion of quality of the interference signals, intended for the suppression of systems with adaptation, can serve as the sum of functionals of the type (1.34) defined for different interference signals. In practice an estimate of interferences to systems with adaptations is most frequently produced with the help of operationally tactical criteria.

An example of a system with adaptations can be the goniometrical coordinator, which switches from target tracking by the active method onto its passive direction finding according to the source of interference radiation located on the target.

The minimax or game systems of automatic control up to now have been studied mainly theoretically. The functioning of certain variants of these systems can be described by the system of differential equations of the following form [13, 14]:

$$\left.\begin{array}{l}\frac{dx_1}{dt}=f_1(x_1, \ldots, x_n; u_1, \ldots, u_r; v_1, \ldots, v_l), \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{dx_n}{dt}=f_n(x_1, \ldots, x_n; u_1, \ldots, u_r; v_1, \ldots, v_l).\end{array}\right\} \quad (1.36)$$

Here $x_1, \ldots, x_n$ are phase coordinates of the controlled object; $u_1, \ldots, u_r$ – useful control of the object carried out by the suppressed side; $v_1, \ldots, v_\ell$ – interference control of the object, carried out by the one creating the interference.

It is assumed that each of the sides knows about the controls of the object carried out by the enemy.

The quality of interference controls can be estimated by a functional of the form

$$Q(u, v)=\int_{t_0}^{T} G(x_1, \ldots, x_n; u_1, \ldots, u_r; v_1, \ldots, v_l)\,dt.$$

In general the suppressed side will try to minimize the functional Q, whereas the one creating the interference will try to maximize it.

Optimum controls of the conflicting sides in a given situation are determined by methods of the theory of games.

1.5. Criterion of Information Loss

Information criteria examined earlier permit estimating the quality of the assigned interference signals, and in many cases it is necessary to estimate by the information criterion the possibilities of concrete means of the creation of interferences or estimate the quality of concrete measures by the organization of interference with the help of the assigned means. The estimates indicated above can be produced with the help of the criterion of information loss.

The measure of information loss can be the volume or area of the space covered by interferences from radar observation. Information loss can also be estimated in relative units as the ratio of volume (area) covered by interferences of space to the whole active volume (area) of space utilized by the operator of the given radar (or radar system).

Let us consider a number of examples explaining the essence of this criterion in reference to radar of circular scanning.

In the absence of organized interferences the operator uses the whole working space within limits of the scanning scale (Fig. 1.2a). The maximum limit of the working region is determined by the ultimate range of action $D_{\text{макс}}$. Disregarding the action of natural interferences and limited resolving power of the radar, it is possible to consider the information loss in this case equal to zero.

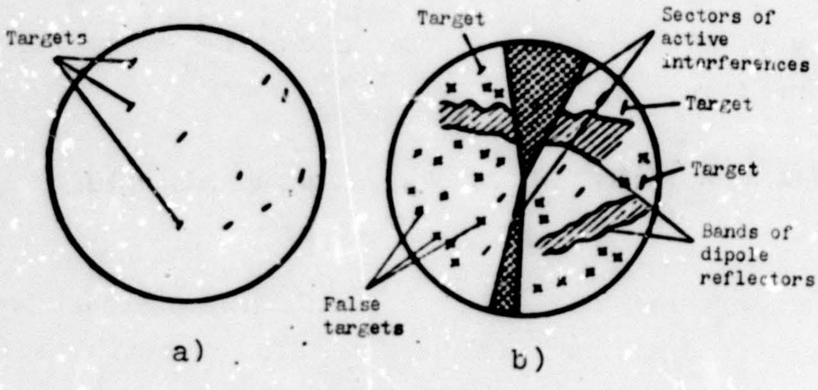

Fig. 1.2. Forms of screens of radar indicators of circular scanning: a) in absence of interferences; b) with the creation of active and passive interferences.

If a sufficiently dense band of passive interferences (dipole reflectors) is created, then within limits of this band the operator is not able to reveal the target (Fig. 1.2b). Consequently, interferences create information loss to the enemy. This loss can be estimated by the magnitude of volume (m^3) or area (m^2) which dipole reflectors occupy.

With the creation of active interferences on the screen of a plan position indicator illuminated sectors will be formed (Fig. 1.2b). The area of these sectors (as compared to the total area of the screen) is also a measure of information loss. Let us note that in Fig. 1.2b false marks are conditionally depicted in the form of small crosses.

It is not difficult to understand that within limits of illuminated zones (Fig. 1.2b)

$$k = \left(\frac{P_n}{P_c}\right)_{in} \geqslant k_n.$$

Equality $k = k_n$ takes place only on the border of the zone.

Borders of the zone covered by interferences are set by power characteristics, and dimensions of the covered region (information loss) depend on parameters of the means of creating interferences by suppressed radar (power of the station of interferences, quantity of dipole reflectors, radar antenna radiation pattern, power of the radar etc.).

1.6. Operational-Tactical Criteria

Depending upon concrete conditions of the combat application for estimating the effectiveness of means and methods of jamming different operational-tactical criteria can be applied. The effectiveness of measures undertaken for jamming in interests of the security of aircraft combat actions for overcoming the PVO of the enemy can be estimated by the numerical value of the probability of overcoming the PVO by strike aircraft or average losses of both strike aircraft and support aircraft.

In solving the problem about the relationship between the number of support aircraft and strike aircraft it can appear expedient to take as a criterion the probability of the fulfillment of the combat mission set before the given group of aircraft. In certain cases the effectiveness of interferences can be estimated by the value of the

mathematical expectation of loss inflicted to the enemy or the prevented damage.

Considering in the first place the aircraft means of RPD, let us examine in somewhat greater detail the methods of estimating the RPD effectiveness according to the probability of fulfillment of the combat mission $P_{бз}$ of the assigned group of aircraft [15].

In order that the combat mission is carried out, there must be a joint offensive of three basic independent events in the first approximation:

- overcoming the PVO by strike aircraft;

- detecting the target;

- applying to the object actions of assigned damage.

In accordance with this

$$P_{бз}=P_{ПВО}P_{ц}P_{ущ}. \quad (1.37)$$

Here $P_{ПВО}$ - probability of overcoming by strike aircraft of the PVO of the enemy; $P_{Ц}$ - probability of detecting the target (object of actions); $P_{ущ}$ - probability of applying to the object of actions of assigned damage.

It is interesting to note that the value of each of the probabilities in the right side of equality (1.37) is determined by the quality of the appropriate electronic security:

$P_{ПВО}$ depends on the quality of information security of the PVO system of the enemy; $P_{Ц}$ depends on the quality of work of onboard detection radar of the strike aircraft; $P_{ущ}$ in many respects is determined by the quality of the control system of the rocket of the air-to-surface class or the radar bombsight.

Estimating the RPD effectiveness organized in the interests of overcoming the PVO, in many cases does not require determination of the full probability of fulfillment of the combat mission. For this it is sufficient to be limited to a consideration of only the probability of overcoming the PVO. The degree of change in the probability of the overcoming of the PVO permits estimating the combat effectiveness of methods and means of RPD in reference to concrete conditions of conducting combat actions and selecting an optimum method of actions for these conditions.

In connection with the application of the probabilistic measure of the effectiveness of measures undertaken for RPD, it is very significant to note the following circumstance. Quantitative values of the probability of overcoming the PVO ($P_{\Pi BO}$), just as all other probabilities entering into (1.37), can be determined only for very concrete conditions of combat application. With a change in conditions of combat application numerical values of corresponding probabilities are necessarily changed.

The organizing combat actions for overcoming the PVO should create interference in order to ensure the maximum value of the probability of overcoming PVO by strike aircraft.

For very particular cases extremely harmfully obtained values of the probability of overcoming PVO are taken as some stable operational-tactical norms. The organization of interferences is in the highest degree a creative operation, and the application of a probabilistic measure for estimating the effectiveness of interferences according to operational-tactical criteria is a more rapid stimulus for investigation of the best decisions than is some stable operational-tactical characteristic.

Considering the influence of subjective factors on the organization of combat actions, one should examine the formula of the type (1.37) more rapidly as qualitative dependences than exact quantitative categories. With the help of a similar form of formulas, as Professor V. V. Shirkov noted, more frequently it is possible to reduce the quantity of words for describing a process than to obtain exact

quantitative results. Nonetheless, the application of quantitative appraisals, including the probabilistic measure, is a progressive matter. It, undoubtedly, stimulates the search for new improved methods of application of interferences and organization of combat actions. If with the appraisal of some variant of combat actions of the losses of aircraft will be great and, accordingly, the probability of fulfillment of the combat mission will be small, it is necessary to search for new methods of conducting combat actions specific for given conditions and the creation of interferences at which the probability of the fulfillment of the combat mission will be the greatest.

The aircraft will overcome the PVO if the joint offensive of three, as a rule, independent events take place: the aircraft is not brought down by fighter aircraft [IA] is not brought down by antiaircraft guided missiles [ZUR], and is not brought down by barrel antiaircraft artillery [ZA].

Consequently,

$$P_{\text{ПВО}}=(1-P_{\text{ИА}})(1-P_{\text{ЗУР}})(1-P_{\text{ЗА}}). \qquad (1.38)$$

Here $P_{\text{ИА}}$ – probability of aircraft destruction by IA; $P_{\text{ЗУР}}$ – probability of aircraft destruction by ZUR; $P_{\text{ЗА}}$ – probability of aircraft destruction by ZA.

In turn, the probability of aircraft destruction by fighter aircraft of the PVO of the enemy is determined by the average number of attacks of fighters $n_{\text{ИА}}$ and by the full probability of aircraft destruction for one attack $P_{\text{ИА}1}$:

$$P_{\text{ИА}}=1-(1-P_{\text{ИА}1})^{n_{\text{ИА}}}. \qquad (1.39)$$

Formula (1.39) determines the probability of the fact that at least one of the $n_{\text{ИА}}$ fighters of the enemy will knock down our aircraft. Quantity $(1-P_{\text{ИА}1})^{n_{\text{ИА}}}$ is the probability of no aircraft destruction by any of $n_{\text{ИА}}$ fighters of the enemy. Analogous formulas can be recorded

for $P_{\text{ЗУР}}$ and $P_{\text{ЗА}}$.

An attack of a fighter will be successful if jointly two events approach:

a) the fighter will be directed by ground means into a region of space near the aircraft, being in which it can reveal the aircraft by its onboard radar and carry out homing guidance;

b) the fighter carrying out homing guidance, will destroy the aircraft by rockets or gunfire.

Accordingly, the full probability of aircraft destruction during one attack of the fighter is determined by the probability of long-range guidance $P_{\text{дн}}$ and conditional probability of destruction $P_{\text{сбит}}$:

$$P_{\text{ИА1}} = P_{\text{дн}} P_{\text{сбит}}. \tag{1.40}$$

Analogous formulas can be recorded for ZUR and ZA.

The quality of the functioning of the system of information security of PVO significantly affects each of quantities $n_{\text{ИА}}$, $P_{\text{дн}}$, and $P_{\text{сбит}}$ determining the combat effectiveness of fighter aircraft of the PVO.

The given considerations permit using as a measure of the operational-tactical criterion the following:

– change in the number of attacks of fighters or launching of rockets according to the assigned target;

– change in the probability of destruction of the aircraft or rocket covered by interferences.

Instead of a change in probability of destruction for estimating the combat effectiveness by the operational-tactical criterion there is frequently used the increase in the miss of a rocket with respect

to the covered aircraft. This is explained by the fact that usually as a result of the action of interferences on the control system by the fighter or the rocket such misses are created, whose value considerably exceeds the guidance errors, which are available in the absence of interferences so that the probability of destruction becomes practically equal to zero. The concept of miss more specifically will be revealed below.

At present the total quantitative methods of estimating the effectiveness of interferences by the operational-tactical criterion in any conditions are still not found. Therefore, it is necessary to estimate the effectiveness for concrete conditions of combat actions.

Since in this book jamming is examined basically in reference to the problem of overcoming aircrafts of the PVO for obtaining recommendations of a general character it is required to expose the most characteristic peculiarities of PVO.

The contemporary system of PVO, irregardless of what theater of operations [TO] it is in, can be represented as the totality of generalized circuits (subsystems):

1) contours of target distribution,

2) contours of long-range guidance,

3) contours of homing guidance.

Functional dependences between parameters of electronic means and indices accepted as criteria of the effectiveness, and in connection with this, an estimate of the RPD effectiveness by operational-tactical crtieria can be obtained for every contour separately.

1.7. Contour of Target Distribution

A simplified diagram of the contour of target distribution is given in Fig. 1.3. The contour includes the following:

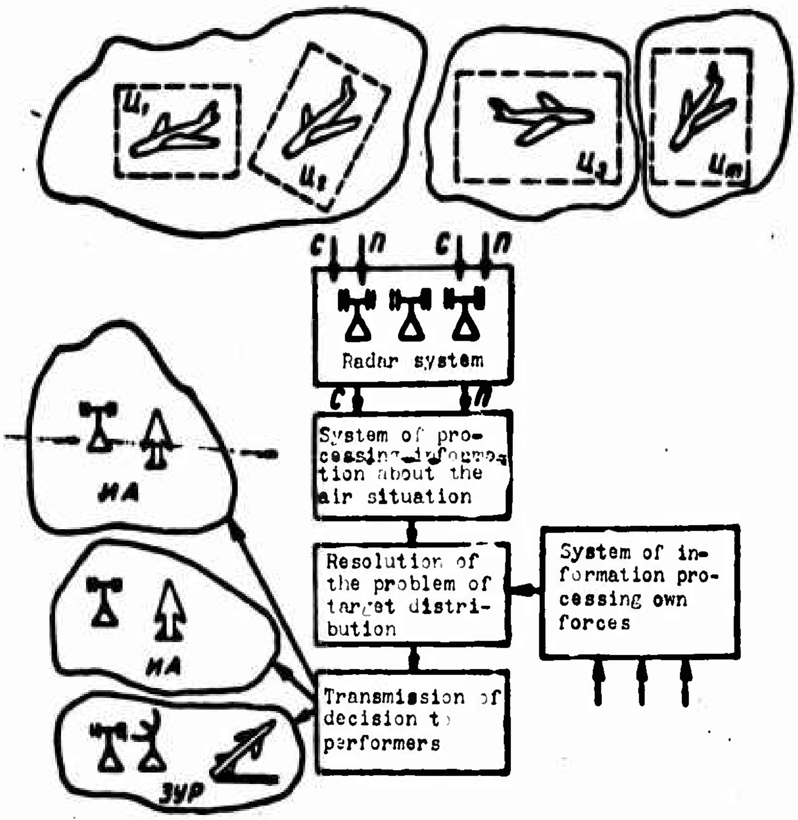

Fig. 1.3. Simplified functional diagram of a circuit of target distribution.

- system of radar (n - radars with the help of which information on m targets is obtained);

- system of processing information about the air situation (this system in an unautomated contour constitutes the service of operators, navigators etc., and in automated - an electronic computer);

- element of resolution of the problem of target distribution, where information is sent about the air situation and the state of inherent forces.

- system of transmission of a decision to the performers.

Conditionally on the diagram the dashed lines show borders within limits of which the position of aircraft $Ц_1, Ц_2, \ldots, Ц_m$ without interferences is determined. Solid lines indicate borders of regions within limits of which coordinates of aircraft under conditions of interferences are determined.

On the basis of an analysis of an air situation it is indicated what targets ($U_1, U_2, \ldots, U_m$) should service the given specific radar from the m available in order of selected target (fighters, ZUR or ZA)[1].

Let us consider to what the creation of interferences (jamming) to the given specific examined circuit will lead.

In combat conditions into each target there should be separated the order of forces for its destruction. In the absence of interferences although the problem of the determination of this order of forces in a specific situation can be complicated it is fully soluble, since the pattern observed on the radar screen corresponds to the real situation of air raid. Quite another position is created with interferences. On radar screens instead of a clear and definite picture of the real air situation there will be seen marks from false targets and even whole sectors and bands from created active and passive interferences (Fig. 1.2b). It is natural that as a result of the comparison of information proceeding from n radar, the enemy succeeds in some degree to comprehend the air situation, but for this time will be lost. As a result of jamming the enemy will have far from a full presentation about the real situation. In the space protected by the given sector or subsector of PVO so-called zones of uncertainty will appear. This will lead to the fact that PVO of the enemy will not know the quantity of attacking targets in each of the regions covered by interferences. Moreover, the enemy must decide whether there are, in general targets in the section closed by interferences and whether this group is demonstrative or strike in order to solve the question of the distribution of the inherent forces.

Thus, as a result of the action of interferences the enemy incorrectly estimates the air situation, in consequence of which there descends the number of attacks or number of launches of controlled

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[1]Below an aircraft overcoming the PVO of the enemy, for convenience, will be frequently called target. It is necessary to remember that this aircraft is a target only for the enemy PVO.

antiaircraft rockets first of all by aircraft of strike forces, in the interests which jamming is created. Furthermore, there can take place a lowering of the average number of attacks by all aircraft of both strike forces and support forces.

Quantitatively the effectiveness of jamming to the system of target distribution according to the operational-tactical criterion is estimated by the change in the number of attacks of fighters or launchings of rockets.

At present there is no general theory which allows finding the change in the number of attacks. However, the following very approximate method of the solution to this problem can be proposed:

1. On the basis of reconnaissance data there is estimated the approximate number of fighters and complexes of ZUR of the resisting grouping of the PVO.

2. The possibility of the complication of the air situation to the enemy is estimated owing to the suppression of his radar of detection, guidance and target designation, which enter into the generalized circuit of target distribution.

3. There is further examined the specific situation with regions, covered by interferences and displaced due to the action of interferences by detection boundaries. In the examination the assumption is made about the fact that the enemy is not able with the help of reserve as means to make the situation clearer and is forced to make the decision on combat actions in accordance with that situation which we create for him with the help of interferences and demonstrative actions.

Putting oneself in the place of the enemy, it is possible to estimate approximately what quantity of fighters will be assigned for actions according to groups of aircraft covered by interferences, and what on the average number of attacks will be apportioned to strike aircraft or support aircraft. Analogously the question is solved

with antiaircraft guided missiles. In a number cases for solving a similar kind of problems methods of the theory of games can be applied.

1.8. Contour of Long-Range Guidances

The contour of long-range guidance[1] starts to function after there is accepted a decision on the target distribution, as a result of which there are assigned specific guidance radar and specific fighters or batteries of ZUR for the action with respect to the target designated by it in the contour of the target distribution. The contour of long-range guidance constitutes, in fact, a servomechanism of the pulse or continuous type (Fig. 1.4).

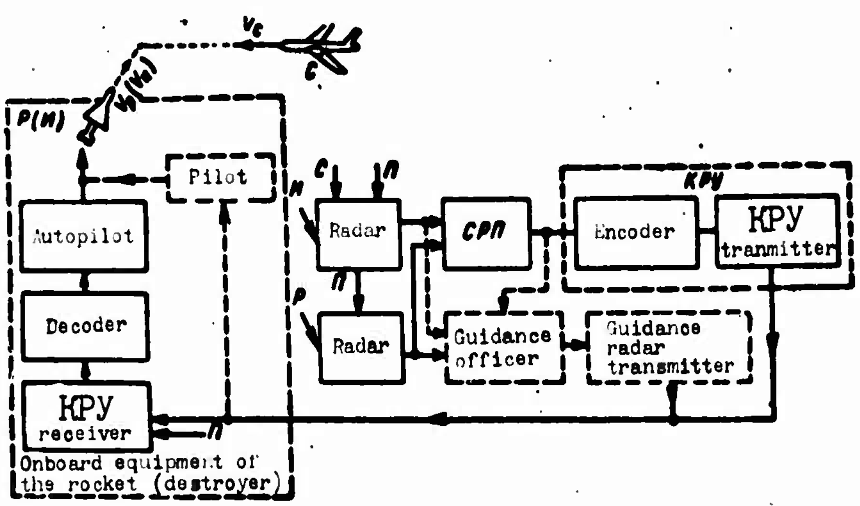

Fig. 1.4. Simplified functional diagram of the guidance contour (long-range guidance).

On the basis of information about the target position data and the destroyer (rocket) and in accordance with the accepted law of guidance with the help of the command radio control link [KRU], the task of the contour to assign the destroyer to such a region from which it could carry out homing guidance with the help of its onboard equipment.

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[1]At present in view of the transient terminology, the terms long-range guidance and guidance usually mean the same process.

In the case of rockets with command guidance not having homing guidance, long-range guidance (or simply guidance) is finished by putting the rocket into the region and blows up, leading to destruction of the target.

For guiding fighters or ZUR at the aircraft (C) one or two radars can be assigned.

The guidance of a fighter (И) or rocket (P) at a target can be produced with the help of a semiautomatic or automatic system. Semiautomatic guidance is applied under the action powerful organized interferences on electronic means entering into the guidance contour. In Fig. 1.4 arrows C and П devote respectively, useful signals and interferences.

Semiautomatic and automatic operating conditions of the guidance circuit to considerable degree differ from each other. Let us examine these conditions in greater detail.

Semiautomatic Guidance of the Fighter (or Rocket)

Detection and guidance radar (one or two radars), operating in conditions of scanning, determines parameters of movement of the target and fighter. Navigator (guidance operator), in watching the radar indicator with the help of a computer [SRP] solves the problem of guiding the fighter (or rocket) to the target. Results of the solution (data for the change in course of the fighter of altitude and speed) are transmitted along communication lines to the pilot, who changes the parameters of movement of the aircraft for going into the assigned position for attack.

A mathematical description of the dynamics of semiautomatic guidance is very complicated due to the fact that the effectiveness of attack depends on a great number of factors, many of which are random and subjective. Considerably complicating the problem more is the fact that contemporary fighters are able, as a rule, to accomplish attack only from the rear hemisphere.

A similar case takes place with completely automatic guidance under the condition that the destroyer is equipped with equipment allowing accomplishment of attack at any attitude.

Automatic Guidance of the Fighter (or Rocket)

Guidance radar, operating in conditions of scanning, gives target position data of the fighter, which enter into a computer. The SRP solves the guidance problem. Results of the solution are transmitted to the fighter or rocket with the help of the KRU. Instructions on the change in course are fed into the automatic pilot, which provides control with respect to course, bank and pitch in accordance with results of solving the concrete problem on guidance. The role of the person (pilot, navigator, operator) in this system is reduced to the control of the efficiency of separate units or the contour as a whole.

In certain systems there is used not one scanning radar but two: one continuously tracks the target, and the second - its own rocket.

The jamming of electronic means by the guidance contour can be provided by changing the information entering into the contour from the radar (radar system) and KRU (radio link).

Active interferences, acting on guidance radar, can create or increase the systematic and random errors, close the contour on a false mark, and in separate cases completely open the contour.

In general one should count on the possibility of an incomplete break in the contour, but only on regular or irregular (accidental) interruptions in the circulation of information.

Passive interferences basically affect resolution of the problem of target distribution. Furthermore, passive interferences can be an effective means for reducing the range of onboard radar of destroyers (rockets) and, consequently, the considerable lowering of the effectiveness of long-range guidance. With the help of the ejection from the aircraft of radar traps, in principle it is possible to provide switching of the contour from the true target to the trap.

With the action of interferences on the KRU can be ensured the suppression of instructions and, which is especially important, the creation of false instructions, which introduce systematic errors into the contour. Interferences to radio control links can also lead to regular or irregular breaks in information circulating in the guidance contour.

Let us estimate the effectiveness of inteferences of the guidance to electronic means. Depending upon the effect to which a specific interference can lead, as a criterion of the effectiveness of RPD, there can be accepted either the value of a miss of the fighter at the time of transition to homing guidance or the probability of guidance under conditions of interferences.

If the application of interferences leads to an increase or creation of systematic errors, then as a criterion of effectiveness it is expedient to take a miss of the fighter (or rocket) Δ taking place at the end of long-range guidance (at the time of transition to homing guidance).

For the criterion of the effectiveness of interferences, the action of which leads to the formation of random errors, it is expedient to take change in value generated by them of the probability of guidance $P_{н}=P_{ДН}$, by which is understood the probability of a hit of the guided destroyer (rocket) into the region of space from which its homing guidance is possible[1].

An Evaluation of the Effectiveness of Interferences with Respect to a Miss (Deterministic Influences)

Usually the guidance of a fighter (rocket) from land (control room) is conducted up to some definite boundary, after which there can be the following two cases:

– the fighter is equipped with radar sight providing the possibility of the functioning of homing guidance circuit, as a result

[1]The evaluation of the quality of guidance with respect to a miss was proposed by A. A. Krasovskiy.

of which there is eliminated the error of guidance or current miss corresponding to it. By current miss we mean the minimum distance, at which the fighter or rocket will fly away from target, if starting from the given moment of time the fighter (or rocket) and target moves rectilinearly and evenly;

– the fighter is not equipped with a radar sight, and the pilot does not visually reveal the target. Therefore, after cessation of the guidance the fighter moves, as a rule, along a tangent to the trajectory of guidance, and the error of guidance (miss, in general, is not selected.

If as a result of the action of the interference on electronic means of guidance the miss Δ at the time of the guidance termination will appear more at the given value of miss Δ_0, selected during the time of homing guidance according the conditions of overloads, then the interference is effective. Otherwise it is considered ineffective.

In the beginning we will examine the simplest scheme of guidance of a fighter or rocket to the target when the guidance of the fighter И to the target Ц is carried out in pursuit and in one plane (Fig. 1.5).

<u>Fighter is equipped with a radar sight</u>. In the absence of RPD it is guided along the calculated trajectory of the $И_0Ц$ (Fig. 1.5).

Let us assume that starting from the moment of time t_0 of guidance circuits start to create interferences to the electronic means, which lead to the formation of deterministic errors in the assignment of the necessary course angle of the fighter (up to t_0 the linear error and current miss corresponding to it are considered equal to zero).

As a result of this the fighter will be guided to the target along a certain trajectory $И_0И_1$, differing from the theoretical $И_0Ц$. At the time of termination of guidance (t_1) the fighter heads for the target with a certain linear error ℓ, which corresponds to the current miss $\Delta_П$.

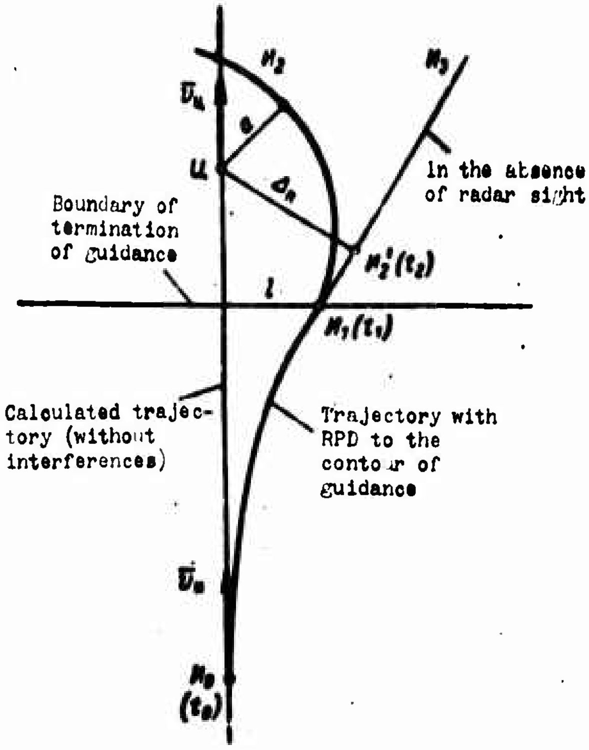

Fig. 1.5. Diagram of the guidance of a fighter or rocket to a target.

Since the fighter is equipped with a radar sight, then at moment t_1 there will be included the homing guidance system, as a result of which the fighter will fly with maximum permissible overload along a new trajectory $\text{И}_1\text{И}_2$, trying to select the linear error ℓ formed with the guidance. If interference is quite effective and leads to the formation of considerable error of guidance ℓ and, accordingly, to considerable current miss Δ_Π, then during homing guidance even with maximum overload it will not be possible to reduce this error to zero, and the fighter will pass from the target (Ц) at a certain distance a, which we will define as the resultant miss.

<u>Fighter is not equipped with a radar sight</u>. Since conditions of homing guidance is absent, then after termination of guidance (moment of time t_1) the fighter, as a rule, moves further along tangent $\text{И}_1\text{И}_3$ toward the real trajectory (Fig. 1.5). At time t_2 the fighter will pass from the target at a certain minimum distance $\Delta(t) = \Delta_\text{п}$, which in this case corresponds to a current miss at the moment of time t_1 and the resultant miss of guidance.

Let us now consider a more common kinematic scheme of guidance of a fighter at a target. We will consider that as a law of control the widespread law of parallel approach is accepted (Fig. 1.6). The kinematic scheme is examined in the relative system of coordinates, the origin of which coincides with the target.

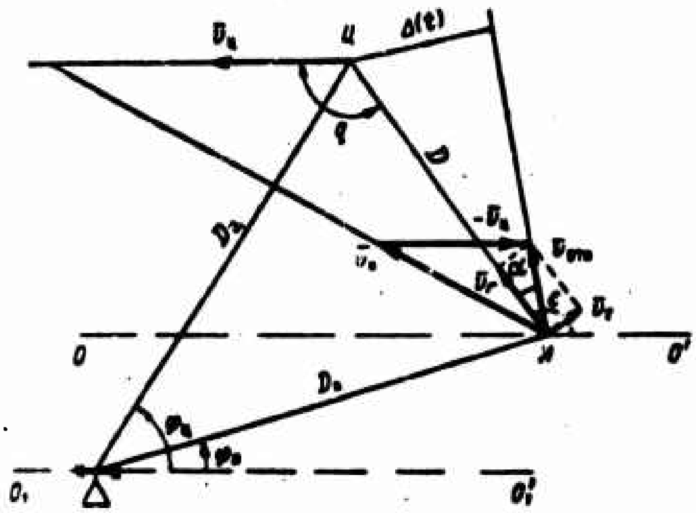

Fig. 1.6. Kinematic scheme of the guidance of a fighter or rocket at a target according to the method of parallel approach.

In Fig. 1.6 the following designations are introduced:

$\bar{v}_Ц$ – speed of aircraft-target; $\bar{v}_И$ – speed of attacking fighter; $\bar{v}_{отн}=\bar{v}_И-\bar{v}_Ц$ – relative speed of fighter in the system of coordinates connected for the target Ц; D – distance between the fighter and aircraft-target at a given moment of time; $D_Ц$ – distance to the target; $D_И$ – distance to the fighter; ИЦ – line of sight of the target; OO' and O_1O_1' – line of the beginning of reading of angles.

With the method of parallel approach control of the course of the fighter is carried out in order to provide condition

$$\dot{\varepsilon}=0, \tag{1.41}$$

where $\dot{\varepsilon}$ – angular velocity of the line of sight of the ИЦ.

Fulfilled condition (1.41) means that the line of sight ИЦ shifts in the process of guidance in parallel to itself. The vector of relative speed $\bar{v}_{OTH}$ is directed along the line of sight, and the fighter, in the accepted relative system of coordinates, will move along the line ИЦ.

With interferences vector $\bar{v}_{OTH}$, as a rule, will not coincide with line ИЦ (Fig. 1.6). Due to this there will appear an error in guidance which, in turn, will lead to a miss. In general the error and current miss corresponding to it will be time functions.

If there are no interferences, then in steady-state operating conditions of guidance the current miss $\Delta(t)$ is small as compared to the maximum permissible for the given system by miss Δ_0, i.e.,

$$\Delta(t) < \Delta_0. \tag{1.42}$$

If effective interferences are created then

$$\Delta(t) > \Delta_0, \tag{1.43}$$

i.e., conditions for disruption of guidance are provided.

Let us set the dependence of miss $\Delta(t)$ on parameters of motion of the aircraft-target and attacking fighter.

The magnitude of the miss $\Delta(t)$ is connected with the value of angle α between the vector of relative speed of the fighter (or rocket) and line of sight by the following simple relation (Fig. 1.6):

$$\Delta(t) = D \sin \alpha. \tag{1.44}$$

Let us decompose the vector of relative speed $\bar{v}_{OTH}$ into two components. One of the components $\bar{v}_r$ is directed along the line of sight, and the second $\bar{v}_t$ is perpendicular to it. Using the connection of linear speed v_t with radius D and angular velocity $\omega = \dot{\varepsilon}$, we will obtain

$$\sin\alpha = \frac{v_t}{v_{отн}} = \frac{D\dot{\varepsilon}}{v_{отн}}. \tag{1.45}$$

Substituting (1.45) into (1.44), we find the desired formula for the current miss [17]:

$$\Delta(t) = \frac{\dot{\varepsilon}D^2}{v_{отн}}. \tag{1.46}$$

Interferences, created to the system of electronic control in principle can doubly affect the change in the current miss $\Delta(t)$. First, directly by means of change in the measured value of angular velocity of the line of sight $\dot{\varepsilon}$ and, secondly, with the help of a change in parameters of proper motion (maneuver of the target) with the electronic control system suppressed by interferences (a break in the guidance circuit by interferences for certain time is considered).

Formula (1.46) serves for determining the current miss (Fig. 1.6), i.e., the minimum distance at which the fighter will pass from the target if starting from the given moment t a uniform and rectilinear motion of the target and fighter (or rocket) takes place.

In real conditions under the action of interferences and maneuver, quantities $\dot{\varepsilon}$, D, $\bar{v}_{OTH}$ will not remain constant. Therefore, the action of interferences and maneuver will lead to a certain integral effect, which can be estimated in the following way.

During the time dt miss $\Delta(t)$ is changed by the magnitude

$$d\Delta(t) = \dot{\Delta}(t)\,dt. \tag{1.47}$$

Accordingly, a change in the miss during time t (the miss accumulated during t under the condition that when t = 0 $\Delta(t)$ = 0) will be

$$\Delta = \int_0^t \dot{\Delta}(t)\,dt. \tag{1.48}$$

Let us differentiate equation (1.46)

$$\dot{\Delta}=\frac{2D\dot{D}\dot{\varepsilon}+\ddot{\varepsilon}D^2}{v_{отн}}-\frac{\dot{\varepsilon}D^2 v_{отн}}{v_{отн}^2}, \qquad (1.49)$$

where $\ddot{\varepsilon}$ – angular acceleration; $2\dot{D}\dot{\varepsilon}$ – Coroilis acceleration;

$$D\ddot{\varepsilon}+2\dot{D}\dot{\varepsilon}=j_n; \qquad (1.50)$$

j_n – component of acceleration of the fighter (or rocket) perpendicular to the line of sight of the target (Fig. 1.7).

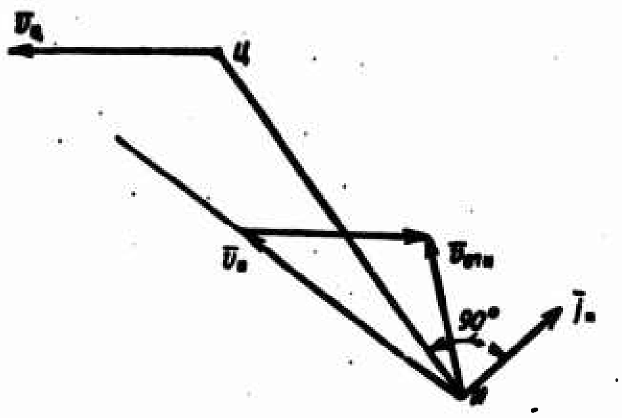

Fig. 1.7. Vector diagram of velocities and acceleration generating the miss with guidance of the fighter.

Taking into account (1.50) and (1.46) the expression for Δ will be recorded in the form

$$\dot{\Delta}=\frac{Dj_n}{v_{отн}}-\frac{\Delta\dot{v}_{отн}}{v_{отн}}. \qquad (1.51)$$

If $v_{отн}=\text{const}$ and $v_{отн}\approx v_{сбл}=\dot{D}$, where $v_{сбл}$ is the rate of closure of the fighter and aircraft-target, then

$$d\Delta(t)=\frac{Dj_n}{v_{отн}}dt. \qquad (1.52)$$

It is possible to present that

$$D=D_0-v_{отн}t, \qquad (1.53)$$

where t - current time; D_0 - initial distance of the guidance.

Substituting (1.53) into (1.52), we will obtain

$$d\Delta(t)=j_n(t_n-t)dt. \qquad (1.54)$$

Here $t_n=\frac{D_0}{v_{отн}}$ - time of guidance.

If the relative speed v_{OTH} is variable with time, then for determination of the miss it is necessary to use equation (1.51), which in this case is reduced to the form

$$d(\Delta v_{отн})=Dj_n dt. \qquad (1.55)$$

Hence

$$\Delta v_{отн2}-\Delta v_{отн1}=\int_0^{t_n} Dj_n dt.$$

In many cases as a result of the action of interferences the fighter or rocket will move with maximum overload

$$j_{n\,макс}=n_{макс}g=\text{const},$$

where $n_{макс}$ - maximum permissible overload; g - acceleration of gravity of earth.

Let us find the miss Δ_{Π} accumulated during the time of action of the interference

$$\Delta_n-\Delta_1=\int_0^{t_n} j_n(t_n-t)dt \qquad (1.56)$$

or

$$\Delta_n=\Delta_1+j_n t_n t_n-\frac{1}{2}j_n t_n^2, \qquad (1.57)$$

where Δ_1 is the initial miss taking place prior to the action of interferences.

If the initial miss is quite small $(\Delta_1 < \Delta_{\text{п}})$ and $t_{\text{п}} = t_{\text{н}}$, then

$$\Delta_{\text{п}} = \frac{1}{2} j_{\text{н}} t_{\text{н}}^2 = \frac{1}{2} j_{\text{н}} \frac{D_0^2}{v_{\text{отн}}^2}. \qquad (1.58)$$

Formula (1.58) also permits finding the miss selected during the time of homing guidance:

$$\Delta_0 = \frac{1}{2} j_{\text{н}} t_{\text{сн}}^2 = \frac{1}{2} j_{\text{н}} \frac{D^2}{v_{\text{отн}}^2}, \qquad (1.59)$$

where $t_{\text{сн}}$ – time of homing guidance; D – distance up to the target at the time of termination of guidance; $v_{\text{отн}}$ – rate of closure of the fighter and target.

It is obvious that RES interference of the guidance of the fighter or rocket will be effective if $\Delta_{\text{п}} > \Delta_0$, i.e., the miss Δ_Π generated by them is greater than the miss Δ_0, selected during the time of homing guidance. The quantitative measure of effectiveness in this case can be the resultant miss

$$a = \Delta_{\text{п}} - \Delta_0. \qquad (1.60)$$

In the case of guidance of the fighter the attack can be considered disrupted if miss Δ_Π will be selected in the very end of the homing guidance, and the pilot will not have time to complete the attack (realization of the launching of rockets or firing of cannons), i.e., one can assume that the attack of the fighter is disrupted if the following condition is fulfilled

$$a = \Delta_{\text{п}} - \Delta_0 \geqslant 0. \qquad (1.61)$$

For the case of ZUR the attack is considered disrupted, if

$$a = \Delta_{\text{п}} - \Delta_0 > R_{\text{п}}, \qquad (1.62)$$

where R_Π – effective destruction radius of guided rocket.

The effective destruction radius $R_{п}$ depends on the type of guided rockets, their combat unit, the relative location of the target and center of explosion etc. [15].

Example of Calculating the Miss

We will consider that as a result of the action of interferences the guidance contour is disrupted for the time $t_{п}$.

If the target after the break in the contour moves evenly and rectilinearly, then the significant miss cannot be accumulated if at the time of the break in the contour the onboard autonomous navigation system will "memorize" the position of the set forward point and with sufficient accuracy will lead the fighter to it. Therefore, it has meaning to estimate the miss in the case when the target carries out a maneuver in speed or direction.

Let us assume that, for example, the target (Fig. 1.8) carries out a maneuver in speed (at the time of the break in the contour it starts to move with acceleration j_c).

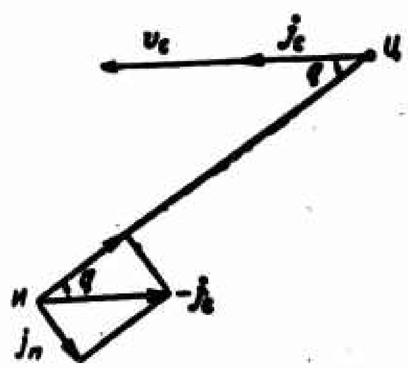

Fig. 1.8. Determination of the component of relative acceleration producting a miss with the maneuver of a target with respect to speed.

Let us determine the component j_n normal to line of sight of the target. From Fig. 1.8

$$j_n = j_c \sin q. \tag{1.63}$$

Substituting (1.63) into (1.58) and considering $\frac{D}{v_{отн}} j_c \cos q \ll v_{отн}$, we will obtain the expression for the miss stored during the time of action of interference $t_{п} = t_{п}$:

$$\Delta_{n} \approx \frac{1}{2} j_{c} t_{n}^{2} \sin \varphi . \qquad (1.64)$$

Estimate of the Effectiveness of Interferences According to the Change in the Probability of Guidance

The formulas mentioned above for misses, from the point of view estimating RPD, give quite reliable results if the values of misses obtained by it considerably exceed the average and mean square values of misses generated by inherent dynamic and fluctuating errors of guidance systems.

With the action on radio electronic means of systems of guidance, many forms of interferences lead to a considerable increase in random errors. Accordingly, the misses will be random.

Above it was indicated that as an operational-tactical criterion of the effectiveness of interferences, the action of which leads to the formation of random errors, it is expedient to assume a change in the value of probability of long-range guidance $P_{\text{нп}} - P_{\text{н}}$.

In order, in general, to be able to estimate the change in probability of guidance P_H under conditions of interferences causing random errors of electronic means (radar of guidance, radio control link), it is necessary to know the law of distribution of errors (misses) and its parameters.

The law of distribution (dispersion) and its parameters can rather accurately be determined with the help of simulation on a computer of processes of guidance under conditions of interferences.

For tentative calculations and estimates it is possible to consider normal the differential law of distribution of misses on one coordinate (Fig. 1.9)

$$p(\Delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\Delta - a)^2}{2\sigma^2}}, \qquad (1.65)$$

where Δ – miss (random variable); a – mathematical expectation of the miss; σ^2 – dispersion of the miss.

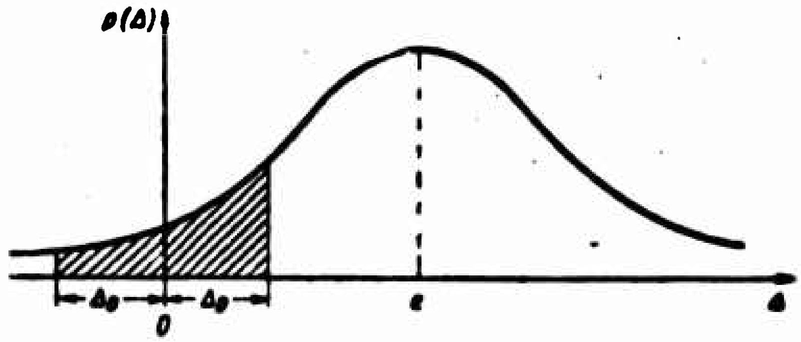

Fig. 1.9. Differential law of the distribution of misses along one coordinate.

The origin of the coordinates in Fig. 1.9 is combined with the position of the attacked aircraft-target.

It is obvious that result of guidance will depend on the magnitude of the miss at the time of the transition of the fighter (or rocket) into homing guidance. If the miss Δ_Π at this instant will be less than the maximum miss Δ_0, selected with respect to conditions of overloads, then guidance can be considered taking place, otherwise it will be unsuccessful.

The probability of guidance of a fighter P_H can be defined as the probability of hit of the random value of miss Δ, defined at the time of transition on the homing guidance, into the interval of misses $(-\Delta, \Delta_0)$, selected by conditions of overloads during the time of the homing guidance[1]:

$$P_H = \int_{-\Delta_0}^{\Delta_0} p(\Delta)\, d\Delta. \tag{1.66}$$

For powerful rockets without homing guidance, on the last stage the probability of guidance can be taken equal in a number of cases

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[1]In general probability P_H should be estimated as the probability of a hit of the random value of a miss into an ellipse depicted in the pattern plane. For a two-dimensional problem it is possible to be limited to the examination of the hit Δ into the interval coinciding with one of the principal axes of this ellipse.

to the probability of destruction $P_{\text{пор}}$. The target is considered destroyed if as a result of guidance of the rocket it falls into the sphere, the radius of which is equal to its effective destruction radius, and the center coincides with the center of gravity of the aircraft (target). Being limited to the examination of guidance in one plane, accordingly we will obtain

$$P_{\text{пор}}=\int_{-R_{п}}^{R_{п}} p(\Delta)\,d\Delta, \qquad (1.67)$$

where R_{Π} – effective destruction radius of the rocket.

In case of rockets, having conditions of homing guidance,

$$P_{\text{пор}}=\int_{-(\Delta_0+R_{п})}^{(\Delta_0+R_{п})} p(\Delta)\,d\Delta.$$

Here Δ_0 – miss selected by conditions of overload during the time of homing guidance.

The presence of conditions of homing guidance provides an increase in the effective destruction radius by quantity Δ_0.

In general, when interferences create systematic and random errors, the probability of guidance P_H in one plane for the fighter is determined by formula

$$P_{н}=\frac{1}{2}\left[\Phi\left(\frac{\Delta_0-a}{\sigma}\right)+\Phi\left(\frac{\Delta_0+a}{\sigma}\right)\right], \qquad (1.68)$$

where a – miss generated by the systematic error;

$$\Phi(x)=\frac{2}{\sqrt{2\pi}}\int_0^x e^{-\frac{t^2}{2}}\,dt$$

– tabulated integral of Gaussian probability (integral of probability).

Graphically the value of probability of guidance P_H is determined by the shaded area under the curve of distribution of misses (Fig. 1.9).

From formula (1.68) it is clear that the value of probability of guidance is influenced by both the mathematical expectation a and dispersion σ^2 of misses.

If as a result of RPD dispersion σ^2 remains constant and the systematic error - mathematical expectation a, is increased, then, as can be seen from Fig. 1.10, the probability of guidance P_H decreases.

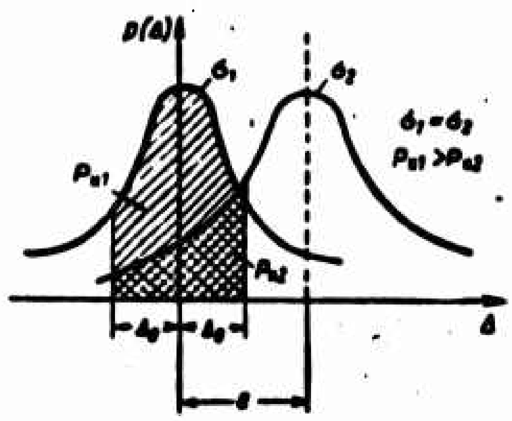

Fig. 1.10. Influence of the systematic error - mathematical expectation of miss a, on the value of probability of guidance.

If with zero mathematical expectation the action of interferences leads to an increase in dispersion of misses, then the probability of guidance P_H also decreases (Fig. 1.11).

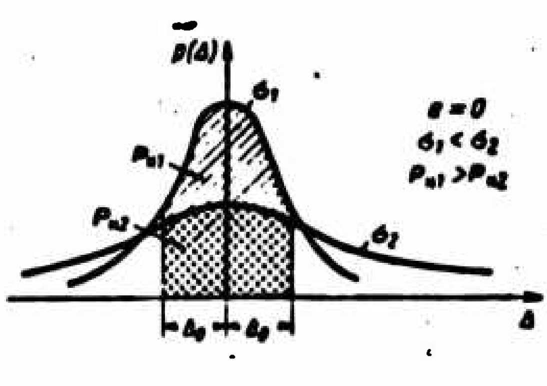

Fig. 1.11. Influence in dispersion of misses on the value of probability of guidance.

In a more general case, when the action of interferences leads to a change in both the mathematical expectation and dispersion of misses, an increase in dispersion can be undesirable (Fig. 1.12).

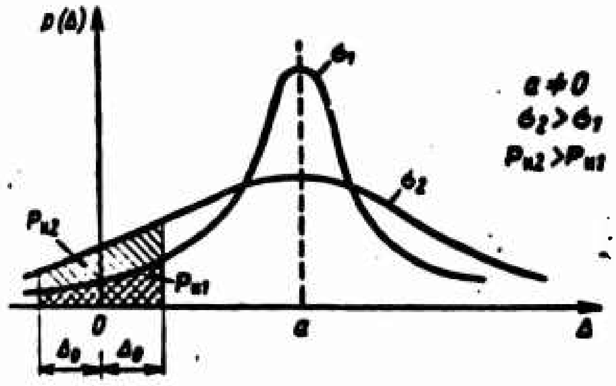

Fig. 1.12. Joint influence of the mathematical expectation and dispersion of misses on the value of probability of guidance.

It is necessary to stress that the methods described above of the quantitative estimate of RPD effectiveness on the guidance contour (miss and probability of guidance) are correct if the assumptions made above are fulfilled:

– the errors of guiding, created with the help of interferences, exceed the natural errors of the guidance system, i.e.,

$$a_\Pi \gg a_c,$$
$$\sigma^2_\Pi \gg \sigma^2_c;$$

where a_Π and σ^2_c – mathematical expectation and dispersion of misses induced by interferences; a_c and σ^2_c – mathematical expectation and dispersion of misses generated by natural errors;

– guidance is carried out in one plane.

1.9. Contour of Homing Guidance

The contour of homing guidance, as a rule, starts to function after termination of long-range guidance.

Homing guidance is necessary for the selection of errors accumulated during the time of guidance.

The fighter (or rocket) passes into conditions of homing guidance after the onboard radar (homing device) "seizes" the target.

The contour of homing guidance, just as the contour of guidance, constitutes a closed servomechanism.

In most cases the homing guidance of rockets is carried out according to the method of proportional navigation (proportional guidance), a particular case of which is the method of parallel approach. For the formation of a command signal with guidance by the method of parallel approach, it is necessary to measure the angular speed of the line of sight $\dot{\varepsilon}$.

One of the possible methods of the formation of the command signal consists in the gyrostabilization in the space of the platform on which the radio link - aircraft radar (homing device) is set. A block diagram of the contour of the homing guidance, in which formation of the command is provided with the help of gyrostabilization in the space of the platform, is shown in Fig. 1.13 [16].

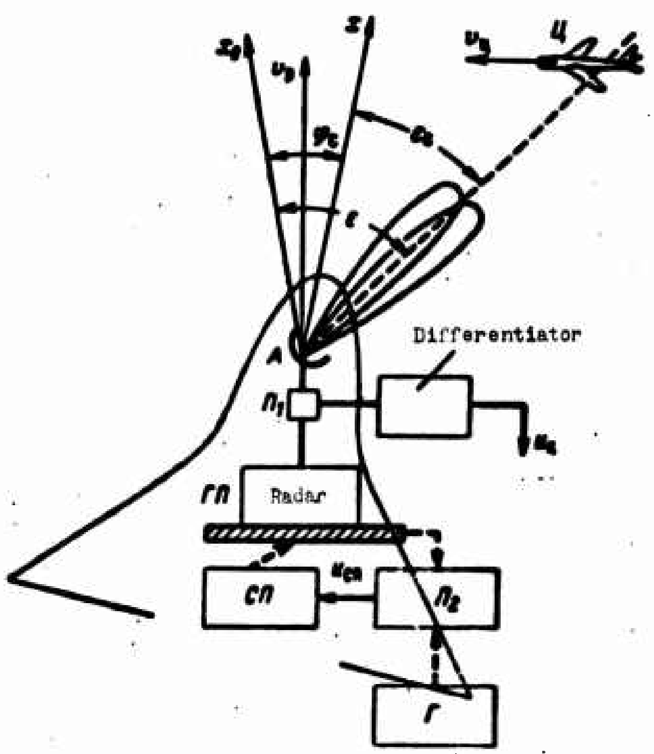

Fig. 1.13. Simplified functional diagram of the contour of the homing guidance with a gyrostabilized platform.

The aircraft radar measures the angle ε_c between axis Ax of the gyrostabilized platform [GP] and direction to the target [ATs]. This angle is the input influence of the system of automatic direction tracking [ASN] of the homing device.

The platform GP is stabilized in space with the help of the actuator [SP] which is controlled by voltage $u_{C\Pi}$ taken from the slider of potentiometer Π_2. The housing of this potentiometer is rigidly joined with the gyrostabilized platform GP and its slider – with the axis of gyroscope Γ, which assigns line Ax_0 motionless in space. With a mismatch ϕ_c between axes Ax and Ax_0 on potentiometer Π_2 there appears the voltage $u_{C\Pi}$, which controls the position of axis Ax.

Thus, the examined system has two circuits of automatic control: internal and external.

The internal circuit carries out gyrostabilization of platform GP so that in an ideal case axis Ax coincides with line Ax_0 motionless in space, and, consequently, $\varepsilon_c = \varepsilon$.

The external (basic) circuit is closed on target Ц and carries out tracking of it. The command signal u_K necessary for guidance by the method of parallel approach is formed by means of differentiation of the voltage taken from potentiometer Π_1, connected with the antenna radar. The servomechanism automatically changes the direction of the velocity vector of the rocket v_p in order to ensure

$$\dot{\varepsilon} = 0.$$

In practice the controlling signal, proportional to the angular velocity of the line of sight of the target, is mostly formed with the help of the rate gyroscope. The latter provides the possibility of the formation of the signal proportional to the derivative of the angle of yaw ϕ of the rocket. The controlling signal of the basic circuit is obtained by means of addition of the signals proportional accordingly to the derivative of the angle of yaw ϕ and derivative of

angle ε_c between the axis of the homing rocket x_{CH} and direction to the target. The aircraft radar in this case is rigidly connected with the housing of the rocket.

A functional diagram, which explains the principle of realization of the system with the rate gyroscope, is shown in Fig. 1.14.

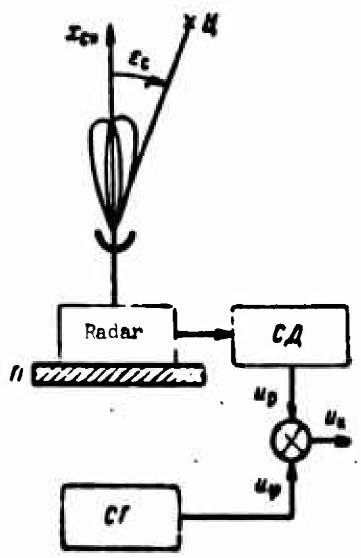

Fig. 1.14. Simplified functional diagram of the contour of homing guidance with a measuring rate gyroscope.

The aircraft radar (homing device) is set on platform П rigidly secured to the housing of the missile. From the output of the radar with the help of a data removal device [SD], having in its composition a differentiator, there is taken voltage u_p, which is proportional to the derivative of angle ε_c, i.e.,

$$u_p = \kappa_\varepsilon \frac{d\varepsilon_c}{dt} = \kappa_\varepsilon \dot{\varepsilon}_c, \tag{1.69}$$

where κ_ε – proportionality factor.

The rate gyroscope [SG], set on the housing of the missile, forms a voltage proportional to the derivative of angle ϕ between the line motionless in space determining the beginning of the reading of the angles and the longitudinal axis of the missile, i.e.,

$$u_\varphi = \kappa_\varphi \dot{\varphi}, \tag{1.70}$$

where κ_ϕ – proportionality factor.

From (1.69) and (1.70) for the examined system we will obtain the relation determining the command signal as the sum of two voltages:

$$u_{\text{к}} = u_{\text{р}} + u_{\varphi} = \kappa_{\varepsilon}\dot{\varepsilon} + \kappa_{\varphi}\dot{\varphi}. \tag{1.71}$$

With guidance by the method of parallel approach there should be fulfilled equality (1.41), which in this case corresponds to $u_{\text{К}} = 0$.

The object of the action of interferences in the homing guidance circuit in most cases is the receiving device of the aircraft radar (homing device).

Active and passive jamming, acting on the circuit through the receiving device of the radar, may cause an increase in random errors in the determination of coordinates and their derivatives and the formation of systematic errors (deterministic influences). Furthermore, an increase in both random and systematic errors in the determination of coordinates can be provided at owing to the periodic or accidental interruptions in the entering of information. This is achieved with the help of interferences leading to the expulsion or withdrawal of gates of circuits of automatic range tracking, speed and angular coordinates.

The quite complete operational-tactical criterion of the effectiveness of interferences to electronic means of the circuit of the homing guidance can serve as a degree of the decrease in conditional probability of destruction.

For approximate calculations it is possible to estimate the probability of destruction as probability of the hit of a rocket in a certain region, which in many cases is a sphere whose radius is equal to the effective destruction radius of the given rocket, and the center coincides with the center of gravity of the aircraft (target).

It is considered that the hit of the rocket in this region ensures destruction of the target with a probability equal to unity, and, accordingly, a miss in the sphere of the indicated radius is equivalent to nondestruction of the target.

Interferences leading to the formation of considerable systematic errors and, consequently, great displacements of the center of dispersion of the rockets can be estimated by the magnitude resultant miss of the rocket (1.60):

$$a = \Delta_{\text{н}} - \Delta_{\text{ф}}. \tag{1.72}$$

If the magnitude of resultant miss is greater than the effective destruction radius of destruction of the rocket $(a > R_{\text{п}})$, the interference is considered effective. This method of evaluation gives satisfactory results when dispersion σ_p^2 of misses of the rocket, generated by fluctuating and dynamic errors of the circuit of homing guidance, is considerably less than a^2.

An estimate of the effectiveness of interferences by the method indicated cannot be conducted if $a^2 \approx \sigma_p^2$. To estimate the effectiveness of appropriate interferences in this case, the application of accurate methods which consider the dynamics of the homing guidance is necessary.

Accurate methods of estimation required is also applied in the case of interferences leading to periodic or accidental short-term interruptions in the entering of information into the contour and also with interferences whose action increases the dispersion of errors owing to the use of nonlinear effects in the contour (limitation by overloads and others).

Let us give certain considerations and formulas for estimating misses of rockets with different forms of interferences.

Interferences to the Goniometrical Channel

Fighter is armed with homing rockets or homing controlled ZUR are applied. In this case under the action of interferences to the goniometrical channel errors in the determination of ε, which are certain $[\varepsilon_{\text{п}}(t) \neq 0]$, time functions will be formed.

Changes of $\dot{\varepsilon}_{n}(t)$, induced by interferences, in turn, generate changes of the current miss Δ. The integral miss is calculated by the formula (1.56).

The resultant miss is determined by formula (1.72) similarly to that as was recommended in the examining of the guidance contour. Let us note that in the case of application of the method of parallel approach, active jamming will be effective only when it provides a change in angular velocity of the line of sight of the target $\dot{\varepsilon}$. Interferences, inducing constant errors in the determination of the angle cannot lead to an increase in the miss of the rocket, since the rocket control signal in this case is proportional not to the angle but the angular velocity $\dot{\varepsilon}$ ($u_{к} = k_{u}\dot{\varepsilon}$, where k_{u} - proportionality factor).

By the moment of the beginning of the action of interference the current miss can be considered approximately equal to zero. During the time of the action of interference $t_{п}$ the miss will be accumulated (at j_{n} = const)

$$\Delta_{п} = j_{п}\left(t_{п}t_{п} - \frac{1}{2}t_{п}^{2}\right). \tag{1.73}$$

If interferences act up to the end of homing guidance (D = 0), then Δ_{0} = 0, and the resultant miss will be

$$a = \Delta_{п} = \frac{1}{2}j_{п\,макс}t_{п}^{2}. \tag{1.74}$$

Usually the interferences act up to a certain minimum distance $D_{мин}$, after which their effectiveness drops and the rocket starts to select the accumulated miss. In this case the resultant miss is estimated by formula

$$a = \Delta_{п} - \Delta_{0},$$

where $\Delta_{0} = \frac{1}{2}j_{п\,макс}t_{сп}^{2}$ - miss selected during the time of homing guidance after termination of the action of interferences; $t_{сп}$ - time remained on homing guidance after cessation of the action of interferences.

Fighter is armed with unguided rockets (guns) or ZUR not having homing guidance is applied. In this case the resultant miss a most frequently equals the miss induced by the application of interferences to the ground guidance system of ZUR or aircraft radar of the fighter[1]

$$a = \Delta_{\text{п}}.$$

Interferences to Range Channel

In contrast to the contour of guidance in the contour of homing guidance interference throught the range channel of the homing device is slightly effective, and in separate cases, in general, ineffective. This is explained by the fact that for homing guidance with a sufficient reserve of flying range the basic is information either about the angular position of the target ε or about angular velocity $\dot{\varepsilon}$. The goniometrical coordinator of the contour of homing guidance can provide the entering of this information with a suppressed channel range.

The source of active jamming can set a course with the same accuracy as that of the target, i.e., it permits carrying out passive homing guidance. However, one should have in mind that if interferences on the range channel lead to a break in contour of angle tracking, then they can be effective.

In the case of the contour of guidance the measurement of range plays an important role, since on land for solving the problem of guidance it is necessary to know angle ε, which is found with sufficient accuracy only with an accurate knowledge of range to the target and to the fighter $D_{\text{Ц}}$ and $D_{\text{И}}$ (Fig. 1.6).

1.10 Optimum Methods of the Application of Interferences

The earlier conducted examination of methods of estimating the

[1]It is considered that the fighter, not being able to correct the trajectory of the homing guidance visually, is forced to fire with unguided rockets or with cannons under jamming conditions.

effectiveness of RPD, as a rule, did not consider any return actions on the part of the enemy suppressed by. In many cases such calculation in principle is necessary. Furthermore, the actual tactics of conducting combat actions by aircraft and PVO proceeds from concrete possibilities of means of RPD and considers these possibilities. Below in specific examples there appear certain methods of the optimization of actions of sides under conditions of interferences.

A direct result of the application of radio interference is the decrease in quantity of useful information proceeding from an electronic device suppressed by interferences, in consequence of which the effectiveness of the application of weapons of destruction decreases (average number of attacks of fighters, average number of launches of ZUR, full probability of defeat after one attack etc.).

The action of radio interference on the suppressed electronic device does not lead to a material destruction of the latter, in virtue of which the enemy can be protected from the interference directly during its action on the device. This circumstance leads to the necessity in the organization of RPD to consider especially thoroughly possible countermeasures of the enemy.

The effectiveness of measures undertaken for RPD is determined by both technical characteristics of applied means of RPD and methods of their application (methods of action).

In carrying out calculations for the optimum application of means of RPD, it is necessary to solve the following group of problems:

– estimate the possible decrease in the quantity of information in control system by means of PVO of the enemy;

– determine the optimum method of the application of interferences (method of actions providing the greatest decrease in effectiveness of the combat application of means of destruction of PVO of the enemy).

Both these problems are mutually connected, inasmuch as the method of application of interferences can considerably affect the quantity of information in the control system. Therefore, in general it is necessary to examine several variants of the application of interferences and for each of them determine its optimum method of actions, after which the best is selected.

Depending upon to what contour of the system of PVO interferences are created, each of the enumerated problems has its own specific character.

Let us consider the peculiarities of calculations for the optimum application of means of RPD to the contour target distribution (Fig. 1.3).

As is known, the mission of the system of target distribution of PVO is to determine the most rational (for a given situation) distribution of aerial targets between units and subunits of fighter aircraft and ZUR.

The main problem of RPD in this case is to make it difficult for the enemy to accept a correct decision on target distribution and thereby decrease the average number of attacks (launches) on strike aircraft.

With the assigned general numerical composition of groups and assigned means of RPD there can be, in principle, many variants of combat formations and variants of the distribution of strike aircraft in regions covered from the observation by interferences. The problem is that of the great number of possible variants of actions select the one at which the average number of attacks (launches) by strike aircraft for assigned conditions of combat actions will be minimum.

Resolution of the problem can be conducted in the following way.

For assigned conditions of combat actions all possible methods of actions of the aircraft are determined. These methods are

conditionally designated and enumerated in a definite sequence (method A_1, A_2, ..., A_m).

All possible methods of actions of the enemy are determined: these methods are also conditionally designated and are enumerated in a definite sequence (B_1, B_2, ..., B_n).

Each method of the actions (A_1, A_2, ..., A_m) is successfully compared to methods of actions of the enemy (B_1, B_2, ..., B_n).

For every comparison, for example A_1 and B_4 or A_1 and B_1 etc., the quantitative measure of effect of the action is estimated.

For the examined problem the quantitative measure of effectiveness is the average number of attacks by strike aircraft.

The results obtained are placed in a general matrix of effectiveness of methods of actions (1.75) to the investigation of which the whole problem is reduced:

A	B			
	B_1	B_2	. . .	B_n
A_1	$\bar{n}_{11}$	$\bar{n}_{12}$		$\bar{n}_{1n}$
A_2	$\bar{n}_{21}$	$\bar{n}_{22}$		$\bar{n}_{2n}$
.	.	.	. . .	.
.	.	.	. . .	.
.	.	.	. . .	.
A_m	$\bar{n}_{m1}$	$\bar{n}_{m2}$	. . .	$\bar{n}_{mn}$

(1.75)

Similar kinds of problems are solved by methods of the theory of games. In the theory of games methods of actions A_1, ..., A_m and B_1, ..., B_n are called pure strategies of sides, and the quantitative measure of effectiveness actions – the payoff. The basis of the method of resolution of problems examined here on the determination of optimum methods of actions is the apparatus of the theory of games.

As was already noted above, to solve the stated problem it is necessary, in the first place, to determine the expected decrease in information in the system of electronic security after which find the optimum method of application of interferences, which lead to minimum losses. Methods of the solution of the first part of the problem are developed in sufficient measure.

The essence of the solution is reduced to the determination of dimensions of regions covered from radar observation by interferences at the assigned quantity of means of interferences (determination of information loss) or to the determination of the necessary quantity of means of interferences (for example, dipole reflectors) for the cover of assigned combat formations. The method of solving similar kinds of problems is discussed below.

However, determination of the quantity of means of interferences necessary to cover the combat formation of aircraft from radar observation does not exhaust the problem, inasmuch as with several regions covered from observation by interferences it is still necessary to place optimally strike aircraft in these regions.

For an explanation let us examine the simplest example. Let us assume that it is possible with the help of an active jammer to create jamming in the space region Π_1, within limits of which not one aircraft $Ц_1, Ц_2, \ldots, Ц_m$ nor one of the k ground radars of the enemy is revealed. Furthermore, let us assume that preliminarily there is set a band of passive jamming (region Π_2) with length L_Π, within limits of which detection of targets is also impossible (Fig. 1.15).

Further we will consider that besides the active jammer the attacking side has two strike aircraft (bombers), and the PVO has only fighter-interceptors.

Let us find the method of distribution of strike aircraft in regions Π_1 and Π_2 at which the average number of attacks by each bomber will be minimum.

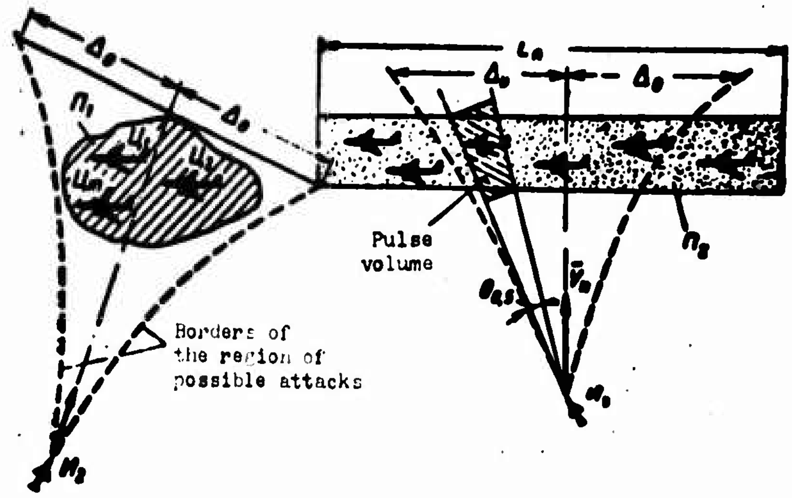

Fig. 1.15. Methods of the application of interferences.

In the assigned conditions the following methods of actions of both sides are possible.

Attacking Side [VVS]:

method of actions A_1 – both bombers in region Π_1;

method of actions A_2 – both bombers in region Π_2;

method of actions A_3 – one bomber in region Π_1, the other – in region Π_2.

Defending Side [PVO]:

method of actions B_1 – both destroyer ($И_1$ and $И_2$) are guided into region Π_1;

method of actions B_2 – both fighters ($И_1$ and $И_2$) are guided into region Π_2;

method of actions B_3 – one fighter ($И_2$) is guided into region Π_1, and the other ($И_1$) – into region Π_2.

A quantitative measure of the effectiveness of actions of both sides in the organization of RPD to the contour of target distribution is the average number of attacks $\bar{n}$ by the bombers. In the examined case

$$\bar{n} = \bar{n}_1 + \bar{n}_2, \tag{1.76}$$

where $\bar{n}_1$ – average number of attacks by the bombers occurring in region Π_1;

$\bar{n}_2$ – average number of attacks by the bombers occurring in region Π_2.

To formulate the initial matrix of the effectiveness of methods of actions, it is necessary that each comparison of methods of actions of the sides be brought into conformity with the average number of attacks by the bomber aircraft.

Assuming that the fighter $И_2$, guided into the group covered with active jamming, can with identical probability attack any of the aircraft of the group, including the one producing the jamming, in first approximation we can assume that

$$\bar{n}_1 = \frac{n_б}{n_б + n_п} n_и. \tag{1.77}$$

Here $\frac{n_б}{n_б + n_п}$ – probability of the selection of the bomber from the total number of aircraft in the region of interferences Π_1

$n_б$ – number of bombers in region Π_1;

$n_п$ – number of jammers in region Π_1;

$n_и$ – number of fighters guided into region Π_1.

Similarly for bomber aircraft covered by passive jamming (region Π_2) and accidentally located in the interference band of length $L_п$

$$\bar{n}_1 \approx \frac{2\Delta_0}{L_\Pi} n_б n_{\Pi 1}, \qquad (1.78)$$

where Δ_0 – maximum miss of the fighter selected according to conditions of overloads during the time of homing guidance;

$\frac{2\Delta_0}{L_\Pi}$ – probability of entrance of the bomber into the region of possible attacks.

This formula is correct for small values of ratio $\frac{2\Delta_0}{L_\Pi}$ and small $n_б$. In general it is necessary to use the more accurate formula

$$\bar{n}_1 = \left(1 - e^{-\frac{2\Delta_0}{L_\Pi} n_б}\right) n_{\Pi 1}$$

The magnitude of the miss Δ_0, selected during the time of homing guidance according to conditions of overloads, is determined by the range of homing guidance D_Π, relative speed of the fighter (or rocket) v_{OTH} and maximum overload j_n. For motion in a horizontal plane

$$\Delta_0 = \frac{1}{2} j_n \frac{D_\Pi^2}{v_{OTH}^2}, \qquad (1.79)$$

where D_Π is determined by the range of the aircraft radar of the fighter taking into account the effect of passive jamming.

The range of the radar on an aircraft flying in a cloud of passive jamming is determined by the effective scattering area [ESA] of the aircraft and by the number of dipoles entering into the pulse volume of the radar. For the case shown in Fig. 1.15 in the first approximation

$$D_\Pi = \frac{k_\Pi \sigma_\Pi v_1 t_\Pi}{\theta_{0.5} \bar{\sigma}_1 n'_c N_{\Pi 0}}, \qquad (1.80)$$

where k_Π – suppression ratio of the radar by passive jamming;

$\theta_{0.5}$ – width of radar beam with respect to half power in radians;

$\sigma_{\text{:}}$ – average ESA of the aircraft;

$\bar{\sigma}_1$ – average ESA of one dipole;

v_{Π} – speed of jammer;

t_{Π} – rate of dropping of reflectors;

n'_{Π} – number of simultaneously dropped packs;

$N_{\Pi\ni}$ – number of effectively acting dipoles in a pack.

Let us assume that L_{Π} = 100 km, D_{Π} = 10 km, j_n = 5 g, v_{OTH} = = 500 m/s. Substituting these data into (1.79), we will obtain Δ_0 = 10 km.

Having all the necessary initial data, with the help of formulas (1.76), (1.77) and (1.78), let us formulate a matrix of results of actions of both sides (matrix of the effectiveness of methods of actions)

A	B		
	B_1	B_2	B_3
A_1	1,3	0	0,6
A_2	0	0,8	0,4
A_3	1	0,4	0,7

(1.81)

An analysis of the matrix does not indicate evident advantages for any of the methods of actions.

The most profitable is method A_2 (bombers in the band of passive jamming), inasmuch as in this case with any methods of the action of the enemy the average number of attacks by the aircraft will not exceed 0.8. With all other methods of actions the number of attacks can be great. For example, with the method of actions A_3 it can be equal to 1.

The method of actions A_3 is less profitable as compared to the method A_2, inasmuch as for two methods of actions of the enemy B_1 and B_2 it gives a considerably large number of attacks by the bombers.

However, if we select the method of actions A_2 as the only one, then knowingly we will enable the enemy to apply method B_2, providing him an average number of attacks by bombers equal to 0.8. Therefore, it is expedient in the examined case to select not any one method of actions but several methods, and the selection of each of them is produced by the random law with a definite frequency (probability). With this the enemy is deprived of the obvious definitiveness in the selection of the method of actions and is compelled to use actions on the assumption of a possible realization by us any of methods A_1, A_2, and A_3.

The probability of the selection of methods of actions should be determined so that the average number of attacks by bombers is minimum. In terms of the theory of games this operation is called the transition from pure strategies to mixed. In the theory of games it is proved that in games of the examined form (matrix game of two players with a zero sum) each of the players has an optimum mixed strategy [15].

The conditionally desired solution will be designated by S. In accordance with what has been said the structure of the solution should be in the following form:

$$S_A=\begin{pmatrix} A_1, & A_2, & A_3 \\ P_1, & P_2, & P_3 \end{pmatrix}.$$

Here A_1, A_2, and A_3 – methods of actions;

P_1, P_2, and P_3 – frequencies of the application of methods of actions A_1, A_2, and A_3.

Let us assume that we found the optimum frequencies of the application of P_1, P_2, and P_3. Then with any method of actions of

the enemy B_1, B_2, and B_3 the average number of attacks by bombers will not exceed a certain number N equal to the number of attacks with the optimum method of actions (in the theory of games N is called the value of the game)[1]. Consequently, if P_1, P_2, and P_3 are optimum frequencies of the application of methods of actions A_1, A_2, and A_3, then following system of inequalities is correct:

$$\begin{aligned} P_1\bar{\pi}_{11}+P_2\bar{\pi}_{21}+P_3\bar{\pi}_{31}&\leqslant N,\\ P_1\bar{\pi}_{12}+P_2\bar{\pi}_{22}+P_3\bar{\pi}_{32}&\leqslant N,\\ P_1\bar{\pi}_{13}+P_2\bar{\pi}_{23}+P_3\bar{\pi}_{33}&\leqslant N. \end{aligned} \tag{1.82}$$

The left-hand side of the first inequality determines the average number of attacks with the method of actions of enemy B_1, the second inequality - accordingly, with the method of actions B_2 and the third - with the method of actions B_3. The inequalities must be recorded because in general not all methods of actions should be applied, i.e., all of them are not always useful.

In those cases when knowingly it is known that all methods of actions are useful, instead of inequalities (1.82) corresponding equalities are written. However, at present this can be stated with full definitiveness for games with the number of strategies at least for one of the sides at not more than two (games 2 × 2 or 2 × n).

The sum of frequencies P_1, P_2, and P_3 is equal to one, i.e., there always takes place the equality

$$P_1+P_2+P_3=1. \tag{1.83}$$

The joint solution of the system of inequalities (1.82) and equality (1.83) exhausts the stated problem.

For the convenience of calculations we will multiply all numbers of the matrix (1.81) by 10, and the value of the game will be

[1]N - least number of attacks which can be achieved in given conditions.

increased accordingly 10 times ($N' = 10N$). However, the desired frequencies P_1, P_2, and P_3 are not changed. Matrix (1.81) will be written now in the following way:

A	B		
	B_1	B_2	B_3
A_1	13	0	6
A_2	0	8	4
A_3	10	4	7

(1.84)

Accordingly, the system of inequalities (1.82) for concrete values of the matrix (1.81) is reduced to the form:

$$\begin{gathered} 13P_1 + 10P_3 < N', \\ 8P_2 + 4P_3 < N', \\ 6P_1 + 4P_2 + 7P_3 < N'. \end{gathered} \qquad (1.85)$$

Further we will divide both sides of inequalities (1.85) and equality (1.83) by N' and introduce the designations:

$$\xi_1 = \frac{P_1}{N'}, \quad \xi_2 = \frac{P_2}{N'}, \quad \xi_3 = \frac{P_3}{N'}.$$

Furthermore, to the left sides of the obtained inequalities there will be added certain nonnegative variables z_1, z_2, and z_3 in order to obtain equalities. Then inequalities (1.85) and equality (1.83) will be converted into the system of equations:

$$\begin{gathered} 13\xi_1 + 10\xi_3 + z_1 = 1, \\ 8\xi_2 + 4\xi_3 + z_2 = 1, \\ 6\xi_1 + 4\xi_2 + 7\xi_3 + z_3 = 1, \\ \xi_1 + \xi_2 + \xi_3 = \frac{1}{N'}. \end{gathered} \qquad (1.86)$$

The problem leads now to the determination of such values ξ_1, ξ_2, and ξ_3 at which value N' will be minimum. Before finding ξ_i ($i = 1, 2, 3$), let us define z_1, z_2, and z_3. These variables can be equal to zero, especially in those cases when all methods of the

actions are useful. The inequality to zero of any variable z_j ($j = 1, 2, 3$) in the examined game indicates inexpediency of the application of any of the methods of actions.

Let us express quantities ξ_1, ξ_2, and ξ_3 in terms of z_1, z_2, and z_3:

$$\begin{aligned} \xi_1 &= -z_1 - z_2 + 2z_3, \\ \xi_2 &= \frac{3}{40} - \frac{3}{5} z_1 - \frac{31}{40} z_2 - \frac{13}{10} z_3, \\ \xi_3 &= \frac{1}{10} + \frac{6}{5} z_1 + \frac{13}{10} z_2 - \frac{26}{10} z_3. \end{aligned} \tag{1.87}$$

Substituting these expressions into the last of equations (1.86), we will obtain

$$\frac{7}{40} - \frac{2}{5} z_1 - \frac{15}{40} z_2 + \frac{7}{10} z_3 = \frac{1}{N'}. \tag{1.88}$$

An analysis of the last equation shows that $z_1 = z_2 = 0$, inasmuch as with an increase in any of them quantity $1/N'$ decreases. With an increase in z_3 quantity $1/N'$ increases, and therefore $z_3 \neq 0$. This circumstance gives the basis to consider that one of the three examined methods of actions is not useful.

In order to expose it, it is necessary to estimate successively the character of the influence z_3 on ξ_i ($i = 1, 2, 3$) and $1/N'$. In this case it is expedient to start the estimate with ξ_3, which determines the frequency of application of method A_3, least effective at the first glance.

Quantity ξ_3 with an increase in z_3 decreases and turns into zero when

$$z_3 = \frac{1}{26}.$$

In order to be convinced of the correctness of the assumption on equality $\xi_3 = 0$ (ineffectiveness of the method of actions A_3), it is necessary to show that quantity $1/N'$ is maximum if $z_1 = z_2 = \xi_3 = 0$.

For this purpose with the help (1.86), let us express 1/N' in terms of z_1, z_2, and ξ_3:

$$\frac{21}{104} - \frac{1}{13} z_1 - \frac{1}{8} z_2 - \frac{7}{26} \xi_3 = \frac{1}{N'}.$$

An increase in any of the variables z_1, z_2, and ξ_3 decreases 1/N', and, consequently,

$$\xi_3 = 0.$$

From the first two equations (1.87) we obtain

$$\xi_1 = \frac{1}{13}, \quad \xi_2 = \frac{1}{8}.$$

From equation (1.88) we find

$$N' = 4{,}95.$$

Further we find the desired frequencies of the application of methods of actions A_1, A_2, and A_3 and the minimum accessible (on the assumption that the enemy acts optimally) number of attacks by bombers:

$$P_1 = 0.38, \quad P_2 = 0.62,$$
$$P_3 = 0, \quad N \approx 0.5.$$

Thus, by applying methods A_1 and A_2 in accordance with frequencies $P_1 = 0.38$ and $P_2 = 0.62$, we provide the lowering of the number of attacks by bombers down to 0.5 irrespective of any possible counter-measures of the enemy in given conditions (certainly, under the condition that he does not apply any other methods of actions besides B_1, B_2, and B_3). In an analogous way the problem on the determination of optimum methods of actions of PVO is solved. The main distinction here consists in the fact that in virtue of the opposition of interests of the sides signs of inequalities are changed and that quantity 1/N' will have to be not maximized but minimized. A corresponding system of inequalities has the form:

$$
\begin{aligned}
q_1\bar{n}_{11} + q_2\bar{n}_{12} + q_3\bar{n}_{13} &\geq N; \\
q_1\bar{n}_{21} + q_2\bar{n}_{22} + q_3\bar{n}_{23} &\geq N. \\
q_1\bar{n}_{31} + q_2\bar{n}_{32} + q_3\bar{n}_{33} &\geq N.
\end{aligned}
$$

Here q_1, q_2, and q_3 – frequencies of the application of methods of actions B_1, B_2, and B_3.

For the examined example $q_1 = 0.38$, $q_2 = 0.62$, $q_3 = 0$, and $N = 0.5$.

The realization of solutions of each of the sides is carried out with the help of random sampling, in particular, a table of random numbers.

For example, if as a result of the solution of the matrix of the game there are found corresponding frequencies of the application of methods A_1 and A_2:

$$P_1 = 0{,}38 \approx 0{,}4 \quad \text{and} \quad P_2 = 0{,}62 \approx 0{,}6,$$

then realization of the solution with the help of two-digit tables of random numbers is carried out by the following method.

A table of random numbers is opened on an arbitrary page. On this page there is selected a number occurring on crossings of lines and columns selected at random. If in the thus found two-digit number the first figure is 0, 1, 2, or 3, then the method of actions A_1 is selected; if, however, this figure is 4, 5, 6, 7, 8, or 9, then the method of actions A_2 is selected.

The application of each of the sides of other methods of actions different from the optimum will lead to great losses if the enemy will act optimally.

In a real situation the number of methods of actions (quantity of pure strategies) for both sides will considerably large.

In general the matrix of the game contains m × n elements (1.75), and the solution of it is reduced to the solution of the following system of inequalities:

$$
\begin{gathered}
P_1\bar{n}_{11} + \ldots + P_m\bar{n}_{m1} \leqslant N, \\
\cdots\cdots\cdots\cdots \\
P_1\bar{n}_{1j} + \ldots + P_m\bar{n}_{mj} \leqslant N, \\
\cdots\cdots\cdots\cdots \\
P_1\bar{n}_{1n} + \ldots + P_m\bar{n}_{mn} \leqslant N, \\
P_1 + \ldots + P_m = 1.
\end{gathered}
$$

For a strict solution of such a system of inequalities methods of a special mathematical discipline – linear programming, are applied. In particular, the example examined earlier was solved by this method. The quantitative solution to problems of linear programming with a relatively small number of lines and columns of the matrix (up to 3-4) can be obtained with the help of comparatively simple means of calculation. With a large number of elements of the matrix the solution in a short period is provided only with the help of a computer.

It is necessary to consider, and this is convincingly shown by an example of the calculation, that solution to the problem on the optimum method of actions cannot be conducted long before the beginning of the combat actions, inasmuch as results of it considerably depend on what methods of actions (what counteraction) must be expected from the side of the enemy. In virtue of this circumstance the solution should be found by taking into account the latest information about the enemy. The more accurately possible methods of actions of the enemy are determined, the more accurate the solution.

In practice a strict solution is not always required. Often it is sufficient to have at least a rough approximation to the optimum. In many cases a rough approximation can be obtained on the basis of only an analysis of the matrix of effectiveness of methods of actions (matrix of the game). An analysis of the matrix consists in a successive examination of it on lines and columns. For the problem of target distribution [matrix (1.75)], with the examination of every line there is written the maximum number of attacks in series for every method of actions $A_1, \ldots, A_i, \ldots, A_m$. From the thus

obtained numbers there is selected the least $\bar{n}_{мин\ макс}$ (in the theory of games this number is called the upper pure value or minimax).

In the analysis of columns in series there is written the minimum numbers of attacks for each of the methods of actions B_1, B_2, ..., B_i, ..., B_n. Selected from these numbers is the largest $\bar{n}_{макс\ мин}$ (in the theory of games this number is called the lower pure value or maximum),

The optimum solution provides a number of attacks of not more than $\bar{n}_{мин\ макс}$, and not less than $\bar{n}_{макс\ мин}$, i.e., the inequality $\bar{n}_{макс\ мин} \leqslant \bar{n} \leqslant \bar{n}_{мин\ макс}$ is always correct.

If $\bar{n}_{макс\ мин} = \bar{n}_{мин\ макс}$, then the methods of actions for which this will take place are optimum, and there is no need to solve the matrix.

For matrix (1.84), for example, results are characterized by the following figures:

A	B			
	B_1	B_2	B_3	min max
A_1	13	0	6	13
A_2	0	8	4	8
A_3	10	4	7	10
max min	0	0	4	

The lower and upper pure values differ considerably, and therefore it is necessary to solve the problem completely.

Obviously, in the case when $\bar{n}_{мин\ макс}$ and $\bar{n}_{макс\ мин}$ do not differ very greatly, one of them can be accepted for the solution. If, however, the difference between the lower and upper pure value is considerable, then, undoubtedly, solution of the matrix is required.

However, even in this case, as a rule, it is possible in the analysis of the matrix to reject beforehand repeated methods of actions (giving approximately an identical effect) and also methods

of actions clearly yielding to at least one of those represented in the matrix.

In certain cases the direct counting of the probability of aircraft destruction in assigned conditions appears expedient.

Given below is a concrete example of the determination of the probability of aircraft destruction under conditions of interferences.

Let us assume that k aircraft fly in series one behind the other in a band of dipole reflectors. The density of the reflectors is sufficiently high so that radar observation of the aircraft in the clouds is excluded. The enemy [PVO] can destroy the first and only the first aircraft. The probability of destroying the lead aircraft by one rocket is considered set and equal to P. If the first aircraft, which is the producer of the band of passive jamming [jammer], is shot down, then its place is taken by the aircraft following behind it, which has the possibility of fulfilling the same function as jammer as that of the preceding aircraft. The probability of destruction of the lead aircraft is identical for all aircraft fulfilling the function of jammers.

It is required to determine the probability of destruction of the k-th aircraft in the group if the enemy can equally produce n rockets. Fire is ceased after the shooting down of the k-th aircraft. The shooting down of the k-th aircraft (event A) can take place as a result of the approach of following incompatible events A_1, A_2, ..., A_{j+1}, ..., A_{n-k+1}.

Event A_1 (the aircraft is brought down after the launching equally of k rockets) can approach only by this method: each of k serially produced rockets strikes the corresponding lead aircraft with probability P. The probability of approach of event A_1 is equal to

$$P(A_1) = P^k.$$

Event A_2 (k-th aircraft is brought down after launching of k + 1 rockets) can approach by C_k^{k-1} methods, the probability of the approach of each of the particular events being equal to $(1 - P)P^k$. The full probability of the approach of event A_2 is determined by the formula

$$P(A_2) = C_k^{k-1}(1-P)P^k.$$

where C_k^{k-1} – number of combinations of k elements with respect to k - 1.

In general event A_{j+1} (the aircraft is brought down after the launching of k + j rockets) can approach by C_{k+j-1}^{k-1} methods. Accordingly, the probability of each particular event is equal to $(1-P)^j P^k$.

The full probability of the approach of event A_{j+1} is written in the form

$$P(A_{j+1}) = C_{k+j+1}^{k-1}(1-P)^j P^k.$$

The full probability of the approach of event A interesting to us (shooting down of k-th aircraft by not more than n rockets) will be defined as the sum of probabilities $P(A_j)$, i.e.,

$$P_n^{\cdot}(k) = P(A) = \sum_{j=1}^{n-k} C_{k+j-1}^{k-1}(1-P)^j P^k.$$

The obtained formula permits estimating the necessary quantity of passive jammers in a column of aircraft, which provides overcoming of the PVO by aircraft of strike groups with an assigned probability.

CHAPTER 2

ACTIVE JAMMING BY RADARS OPERATING IN SCAN CONDITIONS

Radars operating in scan conditions comprise the basis of the system of information security of contours of target distribution. They provide information contours of long-range guidance. Usually the radars, operating in scan conditions, are territorially united into systems and subsystems sometimes called radar field.

In most cases a radar of the examined type operates in pulse conditions. The detection of targets at low altitudes can be provided by radars operating in conditions of continuous radiation. The character of the information loss inflicted by means of active jamming of the radar operating in scan conditions, in general was determined earlier.

2.1. Methods of Estimating Information Loss, Inflicted by Means of Active Jamming

Earlier it was shown that the effectiveness of interferences depends on relationships of powers of the interference and signal, i.e., the interference can inflict a set information loss only under the condition[1]

$$k=\left(\frac{P_{\text{п}}}{P_{\text{с}}}\right)_{\text{вх}}\geqslant k_{\text{п}}, \tag{2.1}$$

[1]One should not confuse coefficients k and $k_{\text{п}}$. The first determines the magnitude of the ratio of the power of interference to the power of the signal, which is obtained for the assigned distance

where $k_{п}$ – coefficient of suppression of the given electronic device by the assigned form of interference; k – ratio of the power of interference $P_{п}$ and signal $P_{с}$ at the input of the receiver.

Coefficient k is a function of parameters of the station of interferences and suppressed electronic device, their mutual location, etc.

To estimate the effectiveness of interferences, it is necessary to set the dependence of the ratio of power of the interference to power of the signal (coefficient k) on parameters of the station of interferences and the suppressed device.

We will consider that two aircraft (jammer PP and covered target aircraft Ц) overcome the antiaircraft defense of the enemy (in this case one radar).

Let us introduce designations for parameters characterizing the station of interferences and suppressed radar (Fig. 2.1).

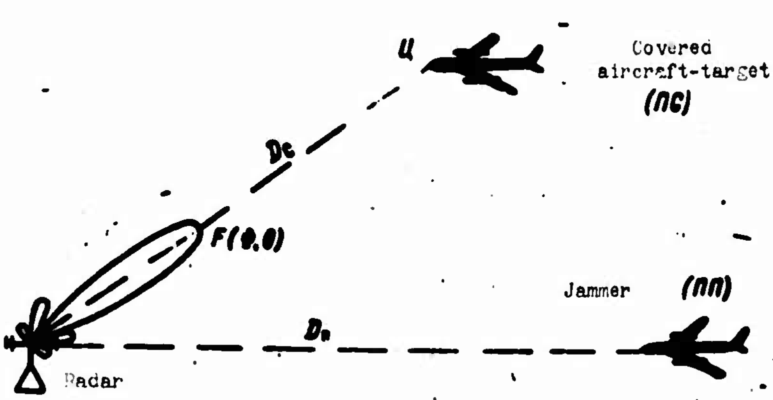

Fig. 2.1. Variant of the creation of active jamming.

Parameters characterizing the system creating interference are the following:

[FOOTNOTE CONT'D FROM PRECEDING PAGE]
between the jamming transmitter and suppressed radar. The second coefficient determines the minimum necessary magnitude of this ratio, which provides a definite information loss.

$P_п$ – power of jamming transmitter;

$G_п$ – maximum directive gain of the antenna of the jamming transmitter taking into account efficiency of the feeder;

$\Delta F_п$ – effective width of the spectrum of interference signal;

$\gamma_п$ – coefficient considering the distinction of polarizations of antennas of the jamming transmitter and suppressed radar;

$\sigma_ц$ – effective scattering area of the covered aircraft (target);

$D_п, \theta_п, \Phi_п$ – polar coordinates of the jammer. Angles $\theta_п$ and $\Phi_п$ are counted off in corresponding planes from the maximum of the antenna radiation pattern of the suppressed radar (Fig. 2.2);

D_c – distance to the covered aircraft.

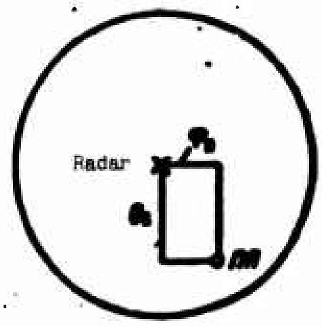

Fig. 2.2. Coordinates of the jammer PP in the pattern plane.

Parameters characterizing the suppressed device are as follows:

P_c – power of suppressed radar taking into account efficiency of the feeder;

G_c – maximum directive gain of antenna of the suppressed radar ($P_c G_c$ is frequently called power potential of the station);

$\Delta f_{пр}$ – transmission bandwidth of the linear part of the receiver of the suppressed radar (it is assumed that $\Delta F_п \geqslant \Delta f_{пр}$);

$F(\theta, \Phi)$ – function describing the standardized antenna radiation pattern of the suppressed radar over the field;

$k_п$ – coefficient of suppression of the given radar by the given interference signal;

A_r – equivalent surface of absorption (absorbing surface) of the antenna of the suppressed radar determined by formula

$$A_r = \frac{G_c \lambda^2}{4\pi}.$$

Let us find the dependence of coefficient k on the enumerated parameters.

The flux density of power of the interference signal at the input of the antenna of the suppressed radar is determined by formula

$$p_{\text{п}}=\frac{P_{\text{п}}G_{\text{п}}}{4\pi D_{\text{п}}^{2}}10^{-0.1\alpha D_{\text{п}}}, \qquad (2.2)$$

where α - coefficient considering attenuation in the atmosphere (dB/km) with passage of the signal tc only one side.

The power at the input of the receiver of the suppressed radar

$$P_{\text{п вх}}=p_{\text{п}}A_{r}F^{2}(\theta_{\text{п}},\Phi_{\text{п}})\gamma_{\text{п}}+P_{\text{ш}}. \qquad (2.3)$$

Here $P_{\text{ш}}$ - power of inherent noises of the receiving device in the passband of the linear part of the receiver

$$P_{\text{ш}}=\kappa TN_{\text{ш}}\Delta f_{\text{пр}},$$

where $\kappa=1.38\cdot10^{-23}$ W/deg·Hz - Boltzmann constant; T - absolute temperature; $N_{\text{ш}}$ - coefficient of noise of the receiver.

Usually the power of the interference signal considerably exceeds the power of inherent noises of the receiver. Therefore, in most cases the second component in (2.3) can be disregarded, i.e.,

$$P_{\text{п вх}}=p_{\text{п}}A_{r}F^{2}(\theta_{\text{п}},\Phi_{\text{п}})\gamma_{\text{п}}.$$

However, in the calculation of the lowering of the range of the radar under conditions of noise interferences, it is necessary to consider inherent noises of the receiver.

Entering into the receiver is only part of the power of interferences, which is determined by the relationship of the width of the spectrum of the interference signal and passband of the suppressed radar reciever. On the assumption of a rectangular approximation of

the spectrum of the interference signal and amplitude-frequency characteristic of the linear part of the receiver of the suppressed device, the power of interference on the input of the receiver within limits of the passband of its linear part is determined in the following way:

$$P_{\text{п вх}} = p_{\text{п}} A_r F^2(\theta_{\text{п}}, \Phi_{\text{п}}) \gamma_{\text{п}} \frac{\Delta f_{\text{пр}}}{\Delta F_{\text{п}}} = \\ = \frac{P_{\text{п}} G_{\text{п}}}{4\pi D_{\text{п}}^2} A_r F^2(\theta_{\text{п}}, \Phi_{\text{п}}) \gamma_{\text{п}} \frac{\Delta f_{\text{пр}}}{\Delta F_{\text{п}}} 10^{-0.1\alpha D_{\text{п}}}. \qquad (2.4)$$

Similarly, for the power of the useful signal at the input of receiver of the suppressed radar it is possible to write

$$P_{\text{с вх}} = \frac{P_{\text{с}} G_{\text{с}}}{4\pi D_{\text{с}}^2} \frac{\sigma_{\text{ц}}}{4\pi D_{\text{с}}^2} A_r 10^{-0.2\alpha D_{\text{с}}}. \qquad (2.5)$$

Substituting (2.4) and (2.5) into (2.1), we find the desired expression for the ratio of the power of interference to the power of the signal at the input of the receiver - coefficient k [suppression ratio]:

$$k = \left(\frac{P_{\text{п}}}{P_{\text{с}}}\right)_{\text{вх}} = \\ = \frac{P_{\text{п}} G_{\text{п}} 4\pi D_{\text{с}}^4}{P_{\text{с}} G_{\text{с}} \sigma_{\text{ц}} D_{\text{п}}^2} F^2(\theta_{\text{п}}, \Phi_{\text{п}}) \frac{\Delta f_{\text{пр}}}{\Delta F_{\text{п}}} \gamma_{\text{п}} 10^{0.1\alpha(2D_{\text{с}} - D_{\text{п}})}. \qquad (2.6)$$

Expression (2.6) is called the equation of antiradar (jamming) for active jamming. It permits finding the ratio of the power of the interference to the power of the signal (coefficient k) depending upon parameters of the suppressed radar, the jamming station and their mutual location.

Figure 2.3 shows the qualitative picture of the dependence of coefficient k on $D_{\text{с}}$ and parameters $D_{\text{п}}$ and $P_{\text{п}}G_{\text{п}}$. As follows from the given graphs, with the assigned power potential of the station of interferences $P_{\text{п}}G_{\text{п}}$ and constant distance to the jammer $D_{\text{п}}$, the suppression ratio (k) at the input of the radar receiver decreases

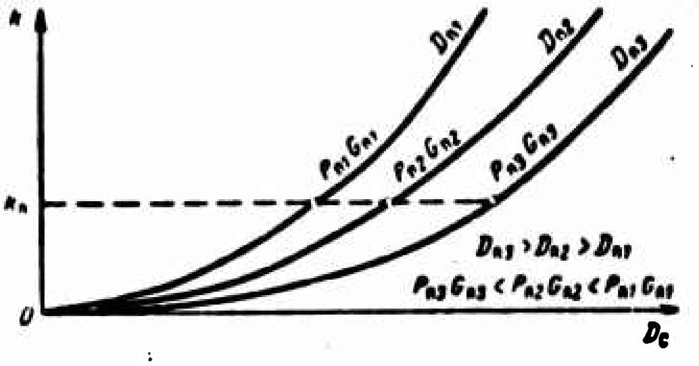

Fig. 2.3. Dependence of the suppression ratio (k) on the distance to the covered aircraft (D_c).

with a decrease in distance to the covered target aircraft.

At a definite distance of the target aircraft from the suppressed radar ratio k will decrease so much that the interference will cease to act. The limit at the approximation to which the interference becomes ineffective is determined by equality

$$k = k_n.$$

The region within which $k = k_n$ (interference is effective) is called the suppression zone.

The suppression zone can be found graphically (Fig. 2.4). For this on the axis of the ordinates it is necessary to plot the quantity k_n and draw a straight line parallel to the axis of the abscissas. The point of intersection of this straight line with curve $k=k(D_c)$ determines the limit of the zone of suppression (Fig. 2.4) along one of the coordinates (range).

As follows from formula (2.6), coefficient k and, consequently, limits of the suppression zone to a considerable degree are determined by the antenna radiation pattern of the suppressed radar. If the jamming transmitter acts with respect to the basic lobe of the antenna radiation pattern, then, obviously, the suppression zone will have a larger extent than in the case of suppression with respect to the lateral lobe.

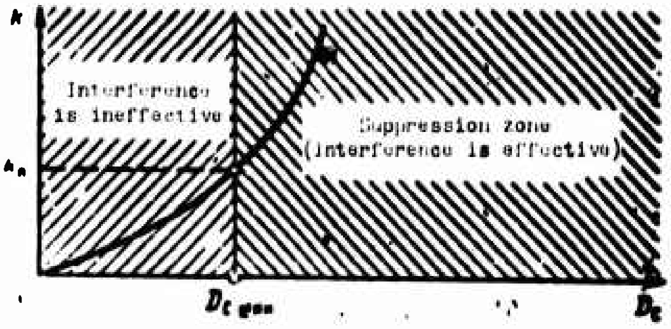

Fig. 2.4. Suppression zone of radar by active jamming.

Figure 2.5 shows in a polar system of coordinates of the suppression zone of the radar with the assigned antenna radiation pattern. It is clear from the figure that at the assigned power potential of the station of interferences and location of the jammer PP relative to the radar, the covered aircraft PS_1 can approach (without danger of detection) in alignment with the jammer (interferences act with respect to the basic lobe of the antenna radiation pattern) much nearer than in the case if PS_2 flies toward the radar not in alignment with the jammer PP (interferences act with respect to the lateral lobes).

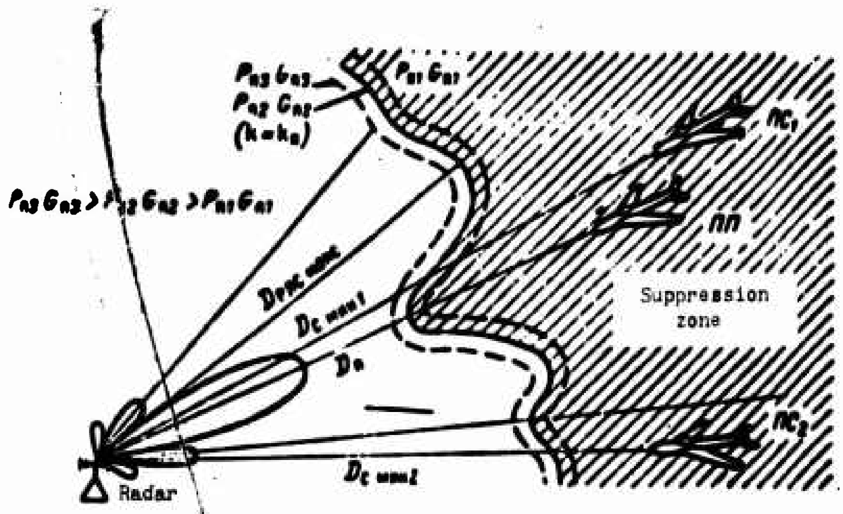

Fig. 2.5. Suppression zones of the radar by the jamming transmitter in a polar system of coordinates.

In other words, the detection range of the first covered aircraft PS_1 will be less than the detection range of the second aircraft PS_2 ($D_{с\,мин_1} < D_{с\,мин_2}$); however, in both cases it is less than the authorized range of the radar ($D_{с\,мин_1} < D_{с\,мин_2} < D_{РЛС\,макс}$).

The increase in power potential of the station of interferences leads to the displacement of the limit of the suppression zone in the direction toward the radar.

In carrying out practical calculations by definition of limits of suppression zones, it is important to know the level of lateral lobes with respect to the basic lobe of the antenna radiation pattern. The level of lateral lobes and their fine structure are an individual characteristic of the radar and depend on the position of the antenna.

For tentative calculations it is possible to take approximately levels of the first and second lateral lobes 20 and 30 dB, respectively, below the level of the basic lobe of the antenna radiation pattern [40].

The case of combination of the jamming transmitter with the covered aircraft is of interest. Formula (2.6) is simplified accordingly and takes the form

$$k = \frac{P_и G_и 4\pi}{P_с G_с \sigma_ц} D_п^2 \frac{\Delta f_{пр}}{\Delta F_п} \gamma_п 10^{0.1\alpha D_п}, \qquad (2.7)$$

If we disregard the absorption of electromagnetic waves in the atmosphere ($\alpha = 0$), then from (2.7) it is possible to find easily the formula for the minimum range of suppression, having placed into it $k = k_п$:

$$D_{п\,мин} = \sqrt{\frac{k_п P_с G_с \sigma_ц \Delta F_п}{4\pi P_и G_и \gamma_п \Delta f_{пр}}}. \qquad (2.8)$$

Sometimes formula (2.8) is called the range formula of the jamming transmitters.

The minimum range of suppression in general can be found from formula (2.6).

With the approach of the jamming transmitter to the radar, the effectiveness of the interferences drops (suppression ratio decreases). This is explained by the fact that in the process of the approach of the aircraft with the jamming transmitter to the radar the power of the signal reflected from the aircraft increases faster than the power of the interference at the input of the radar receiver.

Actually, the power of the signal reflected from the aircraft, is inversely proportional to $D_{п}^{4}$, and the power of the interference signal – $D_{п}^{2}$.

Formulas (2.6) and (2.8) are correct, if the receiver is not overloaded by interference.

The real receivers and indicator devices have a limited dynamic range, so that usually there exists a certain value of the power of interference $P_{п\,макс}$, at which the overload of the receiver approaches, after which it loses the possibility of fulfilling its functions in separating advancing information. Figure 2.6 shows two cases of the amplification of the mixture of the signal and interference. Case a corresponds to such a level of interference $P_{п}$ at which there is no overload of the receiver. The signal is confidently observed against the background of interferences. Case b corresponds to the overload of the receiver by interferences of great intensity. Although the power of the signal is considerably greater than the power of the interference, the signal at the output of the receiver is not observed.

Figure 2.7 qualitatively shows the dependence of absolute values of powers of the interference $P_{п}$ and signal P_{c} and also their ratio k from range $D_{c}(D_{c}=D_{п})$. Plotted on the axis of the ordinates is the value of the suppression ratio $k_{п}$ and, furthermore, the value of maximum power $P_{п\,макс}$ at which the overload of the receiver occurs. Curves of Fig. 2.7 correspond to the case when the overload of the receiver approaches with the power of interference $P_{п\,макс}$ greater than

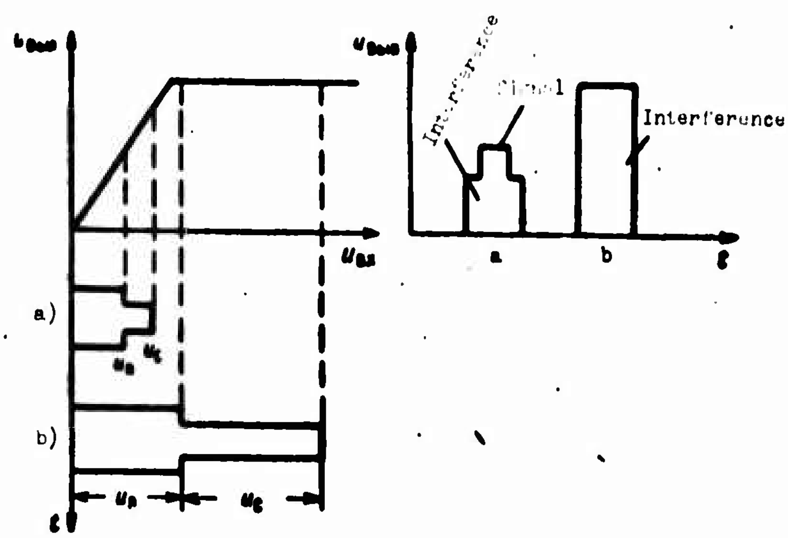

Fig. 2.6. Overload of the receiver by interferences of great intensity: $u_п$ – voltage of interference; u_c – voltage of signal; a) case of the receiver overloaded by interference; b) case when the receiver is overloaded by interference (but $u_c > u_п$).

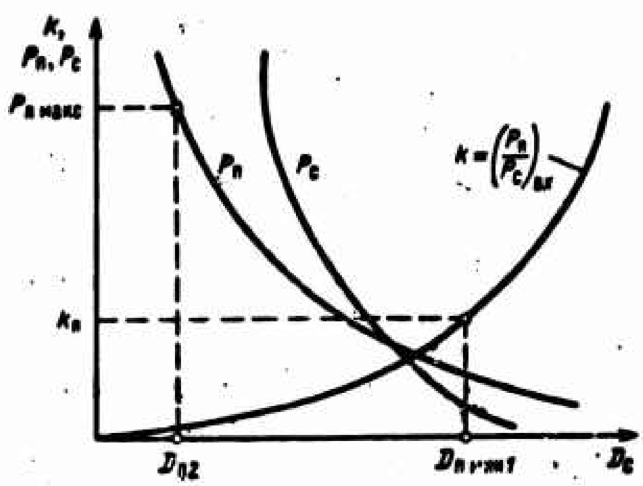

Fig. 2.7. Suppression zones by active jamming of the radar with a limited dynamic range of the receiver.

that which is necessary for suppression at the assigned suppression ratio $k_п$.

Therefore, in the interval of ranges $D_{п\,мин}$ and $D_{в_2}$ the receiver will not be suppressed. However, starting from range $D_{в_2}$ and up to zero ranges, it again will be suppressed but owing to the overload of the receiver-indicator device. In principle there can be the case when $D_{в_2}>D_{п\,мин}$, and then the effectiveness of interferences at short ranges will be greater than at large ranges.

At present in radar measures are taken to weaken the action of strong interferences (limitation, instantaneous automatic gain control [AGC], etc., [20]), and therefore in the determination of minimum range of suppression $D_{п\,мин}$ one should not count on the effect of overload of the receiving device of the radar.

Region of Ambiquity

The concept "suppression zone" pertains to one radar. It is introduced for estimating the action of active jamming in statics. In reality in the overcoming of antiaircraft defense, information on target position data enters into control centers of (guidance) from several radar located in different places. Information about targets and jammers in centers of control is processed, and data of one radar are supplemented and refined with the help of data from other radar. Therefore, in the dynamics of combat the region of the action of interferences (region covered by interferences), in general, will not correspond to the suppression zone.

For example, if data on coordinates of the jammer (PP) proceed from two radars (Fig. 2.8), then as a result of their comparison (analysis) one can determine the position of PP with greater accuracy than in the case of one radar (with two radars the base method of the measurement of range can be applied).

For each of the two radars we have, accordingly, suppression zones determined by areas of sectors S_1 and S_2. A comparison of these zones, besides solving the problem of the measurement of range, permits to considerable extent increasing resolving power of the system of radars under conditions of interferences.

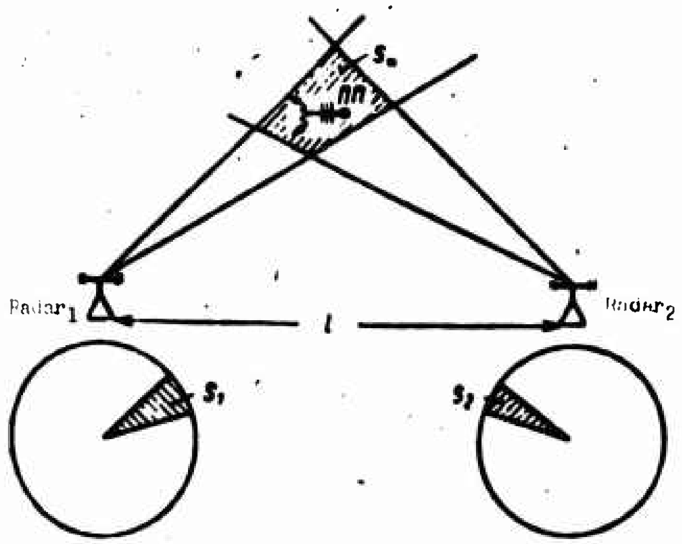

Fig. 2.8. Region of ambiquity forming around the jammer.

The accuracy of determining the coordinates of the jammer and aircraft covered by it depends on the magnitude of sectors S_1, S_2 and magnitude of delay in the entering of information from different radars. In the analysis of data from two radars the accuracy of determining the coordinates of PP will be increased, but nevertheless will remain smaller than in the case of operating without interferences. Thus, the presence of interferences leads to the formation around the jammer of a certain region $S_{н}$, called the region of ambiquity. The dimensions of it are determined by the resolving power and accuracy of the radar system under conditions of interferences.

It is obvious that with several radars

$$S_{н}<S_1, S_2, \ldots, S_n.$$

In a particular case of one radar the region of ambiquity coincides with the suppression zone, i.e.,

$$S_{н}=S.$$

Dimensions of regions of ambiquity very approximately (neglecting the delay) can be found with the help of formulas (2.6), (2.8) and curves shown in Fig. 2.4, 2.5 and 2.7. By knowing the dimensions of

regions of ambiquity and the character of their change with time, it is possible to solve certain jamming problems:

- determine the minimum ranges of suppression;
- find safe sections of the route in the zone of antiaircraft defense;
- calculate the orders of forces and means of jamming necessary for the suppression of a given radar system.

2.2. Continuous Noise Interferences

Radars operating in conditions of scanning, in principle, can be created:

- continuous noise interferences,
- pulse interferences.

Figure 2.9 shows the approximate form of radar screens in the case of the action on them of noise and pulse interferences.

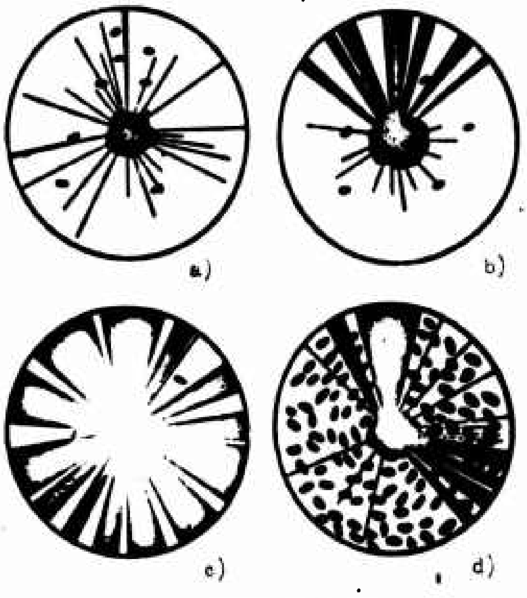

Fig. 2.9. Form of plan position indicator [PPI] of radar with the influence active jamming: a) weak continuous noise interferences; b) interference of average intensity; c) strong interferences; d) action of continuous noise and pulse interferences.

Figure 2.9a corresponds to the influence of weak continuous noise interference on radar. The presence of several bright rays on the screen is explained by the influence of active jamming on lateral lobes of the antenna radiation pattern of the given radar.

An increase in power of the interference at the input of the receiver leads to the expansion of lighted sectors (Fig. 2.9b). Very intense interferences lead to an overload of indicator, as a result of which almost the whole screen of the radar indicator is lighted (Fig. 2.9c). Within limits of the lighted sectors quite powerful interference signals exclude the possibility of radar observation of targets.

A direct result of the action of continuous noise interferences is the camouflage of useful signals in a certain solid angle and corresponding range interval. Due to this the resolving power considerably worsens, and the accuracy of determining the direction toward the target decreases. Measurement of range with the help of radar can in general be excluded over a prolonged time.

Noise interference signals are the most universal among the interference signals known to the present time. They provide the fundamental possibility camouflaging useful signals of any structure and form. If the interference signal constitutes white Gaussian noise, then the probability of correct detection of the useful signal in noises at the output of the optimum receiver is determined only by the ratio of energy signal E to the spectral density of noise G and does not depend on the form of the signal. Moreover, as the theory of the detection of signals in noises shows, the threshold relationship $(E/G)_{\text{пор}}$, which corresponds to the assigned probability of detection with a certain probability of a false alarm, does not depend on the realized method of optimum processing of the signal. Therefore, to create effective noise interferences it is necessary and sufficient to provide only the definite value of the ratio E/G, which corresponds to the permissible value (with respect to conditions of the overcoming of antiaircraft defense PVO) of the probability of correct detection of the useful signal in noises and the assigned

probability of a false alarm.

The coefficient of noise suppression by the interference signal of pulse radar, operating in scan conditions stands for such a value of the ratio of power of the interference signal P_n, within limits of the passband of the linear part Δf_{np}, to the power of the useful signal P_c at the input of the optimum receiver at which the probability of correct detection of the train of n pulses is equal to D = 0.5 with the probability of a false alarm $Q_0 = 10^{-5}$.

In this case the optimum receiver in the sense of Neumann-Pearson is considered, which provides the greatest probability of correct detection D at the assigned probability of a false alarm Q_0. This receiver, in the case of incoherent high-frequency pulses of trains is constructed according to the following block diagram:

- linear part of the receiver, which is the optimum filter for each pulse of the train;
- linear detector;
- post-detector pulse integrator.

The optimum receiver of incoherent high-frequency pulses in practice is realized in the form of a standard superheterodyne receiver with a plan position indicator as the integrator (indicator with afterglow).

The theory of detection determines the dependence of the probability of correct detection D on the ratio of the power of the signal to the power of the noise interference $q=\sqrt{\frac{2,6}{k}}$ and on the number of pulses n in a train with the assigned probability of a false alarm.[1]

[1]The value of the probability of a false alarm in certain cases can vary in comparatively wide limits. Thus, for example, with a change in the probability of the false alarm of 10^6 times, the signal-to-noise ratio corresponding to the probability of correct detection equal to 0.5, changes a total of 1.4-1.5 times.

These dependences for useful signals with a constant amplitude are depicted in Fig. 2.10a. If detection of a fluctuating signal is produced, then it is possible to use the characteristic of detection given in Fig. 2.10b.

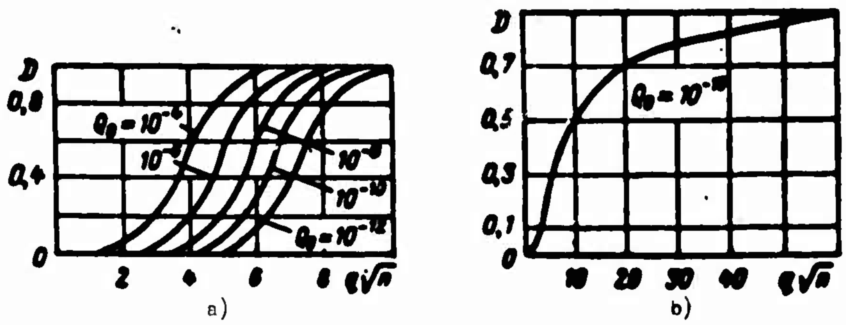

Fig. 2.10. Characteristics of detection: a) deterministic signal; b) fluctuating signal.

Let us assign $Q_0 = 10^{-5}$, then the value D = 0.5 is ensured if

$$q\sqrt{n}\approx 4; \tag{2.9}$$

where

$$q=\sqrt{\frac{2,6}{k}}=\sqrt{2,6(P_c/P_n)_{\text{вх}}}. \tag{2.10}$$

From (2.9) and (2.10) the formula for the suppression ratio is easily obtained by the noise interference signal of pulse radar operating in scan conditions

$$k_n=\frac{n}{6}. \tag{2.11}$$

Let us express the suppression ratio k_n in terms of parameters of the radar: the pulse repetition frequency width of beams of antenna radiation pattern and speed of rotation of the antenna of the suppressed radar.

The number of pulses n in the train is determined by the irradiation time of the target in one scanning cycle and the pulse repetition frequency

$$n=t_{\text{обл}}F_n, \tag{2.12}$$

where $t_{обл}$ – irradiation time of the target; $F_{и}$ – pulse repetition frequency.

Let us find the irradiation time

$$t_{обл}=\frac{\theta_{0.5}}{6N_{об/мин}}, \tag{2.13}$$

where $\theta_{0.5}$ – half-power width of the antenna radiation pattern of the suppressed radar; N – number of revolutions of the antenna per minute.

Taking into account (2.12) and (2.13), from (2.11) we will obtain the desired expression for the suppression ratio $k_{п}$:

$$k_{п}=\frac{F_{и}\theta_{0.5}}{36N_{об}/T_{мин}}. \tag{2.14}$$

Formula (2.14) is obtained on the assumption that the beam of the radar antenna scans only in azimuth and that the interference signal constitutes white Gaussian noise.

White Gaussian noise possesses the greatest camouflaging properties among other random interference signals of assigned power. The real noise interference signals, created with the help of jamming transmitters, according to statistical and spectral characteristics differ from white Gaussian noise in virtue of which they yield to it with respect to camouflaging possibilities.

As was already noted earlier, the measure of camouflaging ability of noise can serve as its entropy or entropy power.

Prior to detection of the target is the a priori uncertainty. The a priori target with a certain probability P_i can appear in each i-th elementary volume (pulse volume) of space serviceable by a certain radar. Designating by A_i the event consisting in the appearance of a target in the i-th elementary volume, it is possible to compose a probabilistic scheme A, which considers the a priori information about the target

$$A=\begin{pmatrix}A_1,\ldots,A_i,\ldots,A_n\\P_1,\ldots,P_i,\ldots,P_n\end{pmatrix},\quad (2.15)$$

$$\sum_{i=1}^{n}P_i=1.$$

The quantitative measure of uncertainty given by the probabilistic scheme is entropy, which is determined by the well-known formula:

$$H(A)=-\sum_{i=1}^{n}P_i\log P_i.$$

If radar operated in the absence of interferences, then as a result of the processing of signals accepted for the cycle (or several cycles) of the scanning the a priori uncertainty would be completely removed, which corresponds to the equality to zero of the a posteriori uncertainty H(B). In this case after the experiment we would obtain exhausting information about the distribution of targets in the serviced space and their coordinates. The quantity of information obtained as a result of the reception of signals is estimated by the quantity of information which in this case is equal to

$$I=H(A).$$

With the creation of interferences to radars after reception of the signals and their processing, the uncertainty is not completely removed. In the first approximation entropy, which corresponds to the a posteriori uncertainty, is equal to the entropy of the actuating noise interference signal H_n. Therefore, under conditions of the action of interferences the quantity of information obtained by radar is equal to

$$I=H(A)-H_n.$$

Thus, the quantity of information obtained by the enemy from the given radar can be decreased owing to the increase in the entropy of the interference signal.

Let us consider several examples of the calculation of entropy of the interference signals.

We will consider that the random variable X is assigned by the one-dimensional density of the probability distribution p(x). The entropy of this random variable can be recorded in the form

$$H(X) = -\int_{-\infty}^{\infty} p(x) \log p(x)\, dx. \tag{2.16}$$

In real conditions the noise created by appropriate devices has limitations both in maximum accessible values and in terms of average power (dispersion).

Let us set the problem to detect for noise interference signals, which are represented by one-dimensional distributions and having an identical limitation for all signals with respect to maximum overshoots or average power, such laws of distribution p(x) to which maximum entropy corresponds (2.16). Let us note that the given problem pertains to the class of the so-called isoperimetric problems of calculus of variations, which are formulated in the following way: among all closed curves with an assigned perimeter find the one which covers the greatest area. The given formulation corresponds to the following analytic recording [21].

Let us assume that there is assigned integral functional

$$\Psi = \int_a^b F(x, p)\, dx, \tag{2.17}$$

where p – a certain desired function of x.

Let us assume that m limitations, superposed on the variable x and function p are also assigned

$$\begin{gathered} \int_a^b \varphi_1(x, p)\, dx = C_1, \\ \int_a^b \varphi_2(x, p)\, dx = C_2, \\ \cdots\cdots\cdots\cdots \\ \int_a^b \varphi_m(x, p)\, dx = C_m, \end{gathered} \tag{2.18}$$

where $\phi_1, \ldots, \phi_m$ – certain assigned functions.

It is required to find such function p(x) which provides the maximum of functional (2.17), taking into account limitations imposed by conditions (2.18).

The given problem in a certain meaning is analogous to the problem for detecting the conditional extremum. Its solution can be obtained with the help of the indefinite Lagrange multipliers.

The maximum of the functional is reached for such p(x) which provide conversion into zero of the following linear combination:

$$\frac{\partial F}{\partial p}+\lambda_1\frac{\partial\varphi_1}{\partial p}+\lambda_2\frac{\partial\varphi_2}{\partial p}+\ldots+\lambda_m\frac{\partial\varphi_m}{\partial p}=0, \tag{2.19}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_m$ – indefinite Lagrange multipliers.

In examined conditions the functional is an integral representation of entropy (2.16). The system of conditions (2.18) is determined by limitations imposed upon noise.

Let us examine the case when noises are limited identically from above and from below, i.e., $-U_0\leqslant x\leqslant U_0$, and can be represented by a one-dimensional distribution. The entropy of the random variable X is equal to

$$H(X)=-\int_{-U_0}^{U_0} p(x)\ln p(x)dx.$$

In all one limitation imposed on p and x is

$$\int_{-U_0}^{U_0} p(x)dx=1. \tag{2.20}$$

Let us find function p(x), providing the maximum H(x).

It is easy to see that for this problem it is possible to write

$$F(x,p)=-p(x)\ln p(x),$$
$$\varphi(x,p)=p.$$

Then equation (2.19) will take the form

$$\ln p(x) = \lambda_0 - 1$$

or

$$p(x) = e^{\lambda_0 - 1} . \tag{2.21}$$

Consequently

$$\int_{-U_0}^{U_0} e^{\lambda_0 - 1} dx = 1 \tag{2.22}$$

or

$$2e^{\lambda_0 - 1} U_0 = 1. \tag{2.23}$$

Substituting (2.21) into (2.23), we obtain

$$p(x) 2U_0 = 1.$$

Hence

$$p(x) = \frac{1}{2U_0}. \tag{2.24}$$

Thus, of all the noises limited from above and from below represented by one-dimensional distribution, the one for which the density of the probability distribution is uniform has the maximum entropy. The value of the entropy of such noise is equal to

$$H_m(X) = -\ln \frac{1}{2U_0} = \ln 2U_0. \tag{2.25}$$

In the second example let us define the law of distribution of the one-dimensional random variable providing the maximum of entropy of noise represented by this distribution and limited in average power (dispersion). The maximized functional will be recorded in the following way:

$$H(X) = -\int_{-\infty}^{\infty} p(x) \ln p(x) dx.$$

The system of limitations has the form

$$\int_{-\infty}^{\infty} x^2 p(x)\, dx = \sigma^2,$$

$$\int_{-\infty}^{\infty} p(x)\, dx = 1.$$

It is obvious that

$$\varphi_1 = p(x),\quad \varphi_2 = x^2 p(x).$$

Fulfilling transformations analogous to those made above, we obtain

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}.$$

Consequently, with the limitation on average power the best noise will be Gaussian noise.

The entropy of the random variable distributed according to the normal law is equal to

$$H_n(X) = \ln \sqrt{2\pi e \sigma^2}.$$

In real conditions the noise voltage inherently both in average power and in terms of maximum overshoots, in virtue of which the optimum distribution will differ both from the uniform and from the Gaussian [22].

Examined above were examples of detecting laws of distribution giving maximum entropy in the class of random processes completely described by the one-dimensional density of distribution. The real interference signal is determined by the assignment of multidimensional densities of the probability of distribution.

The noise, which has limited width of the spectrum F_n and limited duration in time T, can be in accordance with Kotel'nikov's theorem

simply determined with the help of the finite number of values equal to $2TF_n$. If the components of noise (random variables), alternating through time $\frac{1}{2F_n}$, are independent random quantities (the correlation between them is absent), then the entropy of such a process is defined as the sum of the entropies of all $2TF_n$ random variables, i.e.,

$$H_\Sigma = \sum_{i=1}^{2TF_n} H_{ni}.$$

The stationary noise has identical dispersion for all $2F_nT$ components, and therefore for it

$$H_\Sigma = 2TF_nH_n.$$

If the noise is Gaussian, then

$$H_\Sigma = 2TF_n \ln \sqrt{2\pi e\sigma^2}.$$

The results obtained make it possible to estimate quantitatively the masking properties of different noises. For this purpose it is expedient to use these concepts: entropy power and noise factor.

The entropy power of noise with band F_n and duration T means the power of white Gaussian noise P_{n0} with the same band and duration, the entropy of which equals the entropy of the examined noise:

$$H_{\Sigma n} = 2TF_nH'_n,$$

where H'_n – entropy on one degree of freedom of real noise.

In accordance with the determination, the entropy power of the real noise can be found if its entropy is equated to the entropy of Gaussian noise

$$H_{\Sigma n} = 2TF_n \ln \sqrt{2\pi eP_{n0}}.$$

Hence the entropy power of the real noise

$$P_{\text{n}0}=\frac{1}{2\pi e}e_{\text{n}}^{2H'}.$$

The noise factor η_{N} is called the ratio of entropy power of the real noise to its average power, i.e.,

$$\eta=\frac{P_{\text{n}0}}{P_{\text{n}}}.$$

where $P_{\text{n}0}$ – entropy power of the noise; P_{n} – average power of the real noise.

For white Gaussian noise

$$\eta_{\text{N}}=1.$$

For any other noise with the assigned power different from the Gaussian

$$\eta_{\text{N}}<1.$$

The factor η_{N} defined in such a way characterizes the quality of the masking properties of noise. Sometimes the noise factor η is determined by experimental means by estimating the values of the suppression ratios of the specific device to white Gaussian noise and certain assigned real noise.

In this case the factor η is found as the ratio

$$\eta=\frac{k_{\text{n}0}}{k_{\text{n}}}, \tag{2.26}$$

where $k_{\text{n}0}$ – suppression ratio under the action of white Gaussian noise, k_{n} – suppression ratio under the action of real noise.

It is natural that suppression ratios are determined under identical conditions on the basis of the same criterion of acceptance of the optimum solution.

Taking into account (2.26) the formula for the suppression ratio by noise interferences (2.14) takes the following form:

$$k_{\text{п}} = \frac{F_{\text{п}} \sigma_{\text{э,ц}}}{36 N_{\text{ш}} f_{\text{имп}}} \frac{1}{\eta}.$$

2.3. Peculiarities of the Radar Suppression with Continuous and Quasi-Continuous Radiation (Narrow-Band Radar)

The ratio of powers of the interference and useful signals at the input of a suppressed radar pulse $k_{\text{н}}$ within limits of the pass-band of the linear part of the receiver is determined by the antiradar equation

$$k_{\text{н}} = \frac{P_{\text{п}} G_{\text{п}}}{P_{\text{с и}} G_{\text{с}}} \frac{4\pi}{\sigma_{\text{ц}}} D_{\text{ц}}^2 \frac{\Delta f_{\text{пр и}}}{\Delta F_{\text{п}}} \gamma_{\text{п}},$$

where $P_{\text{с и}}$ – pulse power of the radar; $\Delta f_{\text{пр и}}$ – passband of the linear part of the receiver of pulse radar.

Let us assume that the pulse radar is converted into conditions of continuous radiation without a change in average power of the transmitter and all remaining parameters with exception of the pass-band of the receiver, which becomes narrow with the transition into conditions continuous radiation in an appropriate way.

Let us estimate the ratio of powers of the interference and desired signals at the input of the radar, considering parameters of the transmitter of continuous noise interferences to be constant.

Before making such an estimate let us note that the narrowing of the passband of the receiving device of the radar of continuous radiation can be performed up to a certain minimum permissible value, determined by the expected character of the a priori unknown changes in parameters of motion of targets serviced by the given radar. These considerations determine the minimum bandwidth of the transmission of one elementary link (one "tooth") of the optimum comb filter.

Assuming that in pulse radar optimum filtration of the pulse train is provided, it is possible to take

$$\Delta f_{\text{пр и}} = Q \Delta f_{\text{пр н}}.$$

Here Q - off-duty fact of pulse radar, which is approximately equal to the number of spectral lines in the main part of the spectrum of the sequence of square pulses; $\Delta f_{\text{пр н}}$ - minimum permissible passband of the linear part of the receiver of the radar of continuous radiation or, accordingly, one elementary "tooth" of the optimum filter of the pulse radar; $\Delta f_{\text{пр и}}$ - minimum permissible passband of the linear part of the receiver of the pulse radar equal to the sum of passbands of separate "teeth."

In accordance with the accepted assumptions the powers of the pulse radar and radar of continuous radiation are connected with each other by the analogous relation

$$P_{\text{и}} = P_{\text{н}} Q.$$

If one were to consider further the transmitter of continuous noise interferences to be the selective [spot] with respect to the spectrum for pulse radar, i.e., to consider $\Delta F_{\text{п}} = \Delta f_{\text{пр и}}$, then the antiradar formula in reference to the radar of continuous radiation will be thus written:

$$k_{\text{н}} = \frac{P_{\text{п}} G_{\text{п}}}{\frac{P_{\text{с н}} G_{\text{с}}}{Q}} \frac{4\pi}{\sigma_{\text{ц}}} D_{\text{н}}^2 \frac{\gamma_{\text{п}}}{Q} = \frac{P_{\text{п}} G_{\text{п}} 4\pi D_{\text{н}}^2 \gamma_{\text{п}}}{P_{\text{с н}} G_{\text{с}} \sigma_{\text{ц}}},$$

where $k_{\text{н}} = \left(\frac{P_{\text{п}}}{P_{\text{с н}}}\right)_{\text{вх}}$ - ratio of the power of interference to power of the signal at the input of the receiver of the radar of continuous radiation within limits of the passband of its linear part.

Considering that in this case $\Delta f_{\text{пр н}} / \Delta F_{\text{п}} = 1$, we will obtain

$$k_{\text{н}} = k_{\text{и}},$$

i.e., in other words, the ratios of powers of interference and useful signals within the passband of the linear part of the optimum receiving devices are identical for pulse radar and radar of continuous radiation if the jamming transmitter is selective over the spectrum for pulse radar, and if the average powers of the radar are equal. If, however, the width of the spectrum of interference is changed so as to provide selectivity of the transmitter over the spectrum accordingly for each radar, then with constant average power of the jamming transmitter and with the equality of the average powers of both radars

$$k_{н} = Qk_{нп},$$

i.e., noise-to-signal ratio for continuous radar is increased Q times.

We made comparison a comparison of the radar of continuous radiation with the idealized pulse radar. As applied to real pulse radar with the incoherent processing signals, the expressed considerations can be referred to with great reservations. The suppression ratio by noise interferences of radar of continuous radiation can be determined with the help of the Neumann-Pearson criterion of detection is similar to that which was done for pulse radars. It is obvious that the suppression ratio of radar of continuous radiation by noise interference signals of an assigned spectral density will be equal to the suppression ratio of pulse radar if the equality of energies signals and their identical processing in the receiver take place. In particular, with the equality of average powers of radar of pulse and continuous radiation, the suppression ratios by their fluctuating noise signal are equal if the parameters of scanning are identical.

2.4. Peculiarities of the Suppression of Broad-Band Radars with Coding

Broad-band radars with coding appeared in connection with the problem of increasing the resolving power in range with a simultaneous increase in preservation of the detection range of small-size targets.

An increase in resolving power in range can be attained in principle by two methods.

The first classical method is based on the decrease of duration of the main pulse $\tau_{и}$. However, this method has a considerable deficiency lying in the fact that for preservation of the range of radar with a decrease in $\tau_{и}$ it is necessary to increase the pulse power of the radar $P_{и}$. An increase in pulse power conflicts with the fundamental and technical difficulties, connected with generation and transmission of high-frequency electromagnetic oscillations of high power. At present the limit of the pulse power is $P_{и}$ orders of several tens of megawatts.

The second method of increasing the resolving power is based on the special coding of the radiated pulse relative to the long duration $\tau_{и}$ and optimum processing of the received signal in the radar receiving device, which provides compression of this pulse to the duration $\tau_{ик}$ (Fig. 2.11).

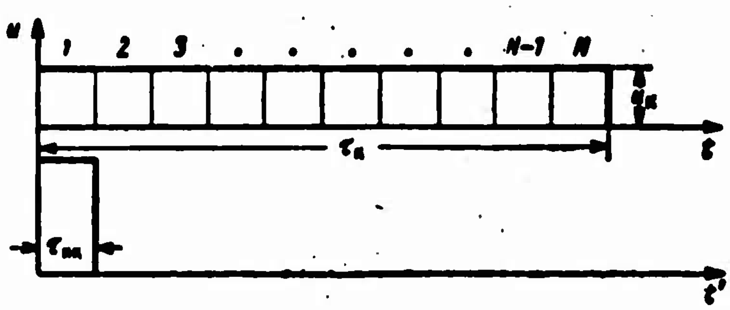

Fig. 2.11. Pulses at the input ($\tau_{и}$) and output ($\tau_{ик}$) of the optimum radar receiver with the coding of signals.

In the system with the compression of pulses there is generated and is transmitted a coded pulse having a duration $\tau_{и}$ and width of the spectrum of frequencies $\Delta f_{и}$, where $\tau_{и}\Delta f_{и} \gg 1$. After the appropriate processing in the receiver there appear short pulses with a duration of $\tau_{ик} = \frac{1}{\Delta f_{и}} \ll \tau_{и}$.

The pulse with a duration $\tau_{\text{и к}}$ at the output of the receiver determines the resolving power of the radar. The energy of the signal is determined by the pulse power of the radiated pulse and its duration $\tau_{\text{и}}$. Thus possibility of increasing the energy of signal is ensured owing to the increase in pulse width $\tau_{\text{и}}$ without impairment of the range resolving power.

Ratio $N=\frac{\tau_{\text{и}}}{\tau_{\text{и к}}}$ is called the compression ratio.

It is necessary to consider that with the assigned average power of the radar transmitter, the second method of increasing the resolving power of the radar (coding) does not lead to any special power gains as compared to the first (proportional increase in power with a decrease in pulse width). Moreover, due to losses with compression in the second case there will take place a certain power loss as compared to the direct method – increase in energy of the pulse signal. However, owing to the possibility of the considerable expansion of the spectrum and increase in average power, broad-band radars have a higher noiseproof feature.

An example of similar kinds of devices can be the system with an intrapulse linear frequency modulation [25]. In this system the carrier frequency of the radiated pulse of duration $\tau_{\text{и}}$ is changed according to the linear law in a certain frequency range f_1-f (Fig. 2.12). On the receiving side the signal is passed through the optimum filter, which possesses dispersion properties. An example of such a filter in principle can be a waveguide, for which, as is known, the group (phase) propagation velocity of wave $v_{\text{гр}}$ depends on the frequency.

The dispersion characteristic of the filter, shown in Fig. 2.12b, provides a more rapid passage (less than the time of the lag) of high frequencies of the signal spectrum. In combination with the law of the change in carrier frequency of the generated pulse, shown in Fig. 2.12a, this allows in principle the possibility of compression of the pulse at the output of the filter (line) up to a certain duration $\tau_{\text{и к}}=\tau_{\text{и}}/N$, where $N>1$.

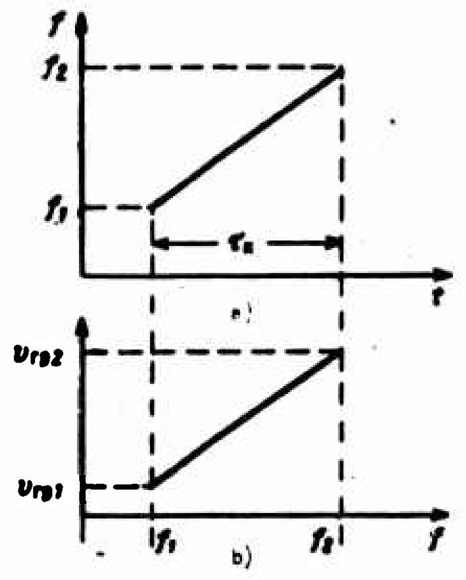

Fig. 2.12. Law of the change in carrier frequency of the main pulse (a) and dispersion characteristic of the optimum radar filter with intrapulse modulation (b).

Let us show that with the assigned spectral density of the noise interference signal, the ratio of the power of the useful signal to the power of noise at the output of the linear part of the receiver remains identical for the standard pulse radar and broad-band radar with coding if their average powers are identical and resolving powers are identical.

We will limit ourselves to an examination of the case simplest for analysis of the phase-keyed signal (Fig. 2.13a). If the signal with amplitude u_{κ}, manipulated in phase, as is shown in Fig. 2.13a, is fed on delay line with N = 7 taps, into some of which (4, 5, and 7) phaseinverter circuits are included, then in virtue of the coherence of signals and their cophasing only during time τ_{κ}/N at the output of the adder (Fig. 2.13b) connected to these taps in first approximation will we obtain a pulse with a duration of τ_{κ}/N and with amplitude $u_{\kappa}\sqrt{N}$. After the adder the convolute pulse [Translator's note: The term "convolute" is not verified] joins the input of optimum for the given pulse of the filter. It is not difficult to note that as a result of the examined transformation, the energy of the convolute pulse realized in the resistance of 1 Ω is equal to the energy of the input signal

$$E=u_{\kappa}^{2}\tau_{\kappa}/2.$$

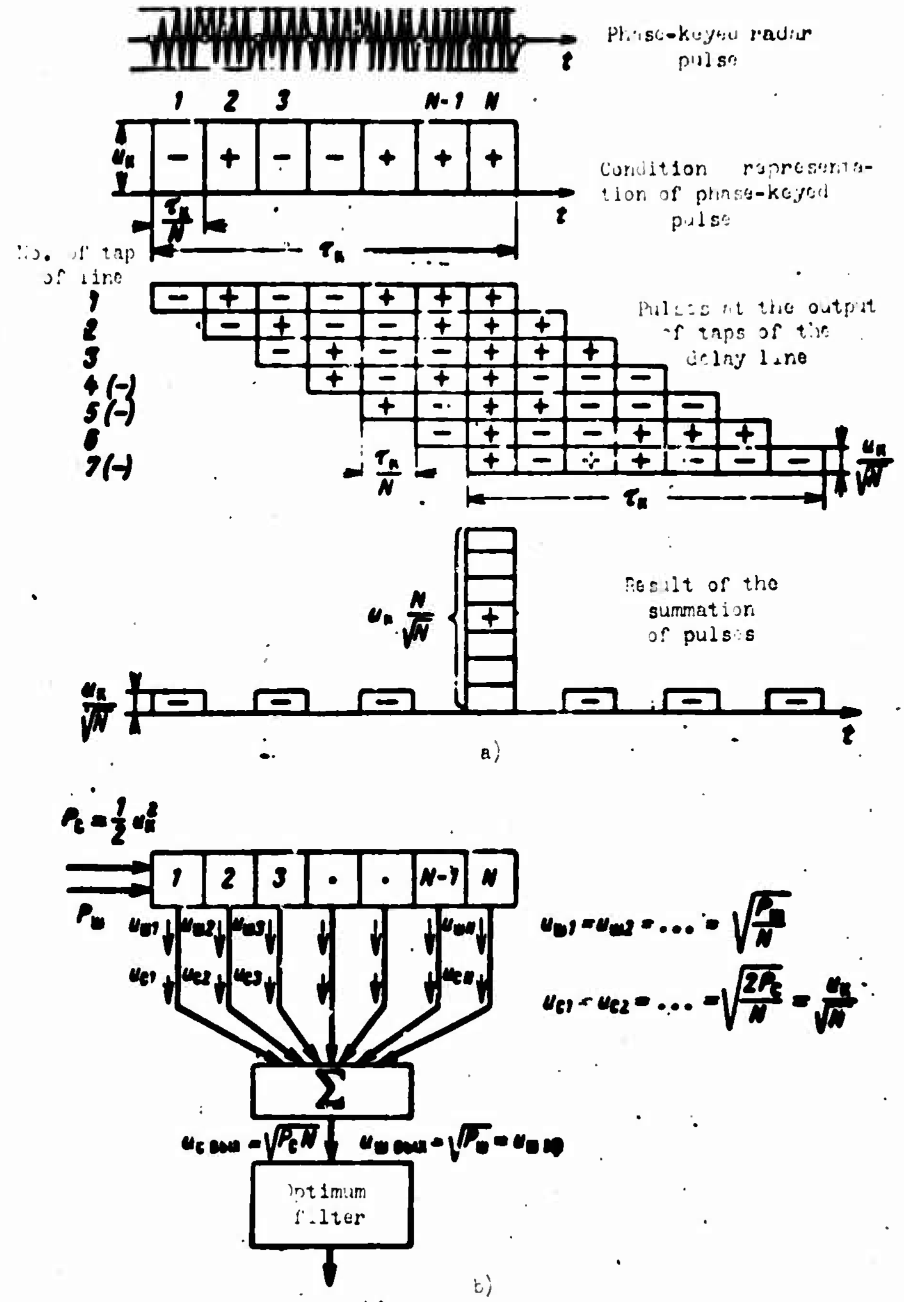

Fig. 2.13. Diagram of the process of treating pulses in an optimum receiving device (a) and functional diagram of the device for compression (b).

The power in the convolute pulse is equal to $u_{\text{к}}^2 = N/2$. The pulse of precisely the same power and duration (energy) can be obtained with the assigned average power directly by increasing the off-duty factor of the radar N times owing to the corresponding decrease in pulse width and increase in its amplitude. The pulse width must decrease N and its amplitude increase $\sqrt{N}$ times.

Thus, in a certain case pulses of identical duration proceed into the input of the optimum filter included after the adder. Inasmuch as the band of optimum filter in both cases is the same, then in virtue of the incoherent addition powers of noises and ratios of power of signals to powers of noises at outputs of the filters will be identical. Hence there directly follows the conclusion concerning energy equivalence with respect to the suppression ratios by noise interferences of radar with coding and the standard radar pulse if they have identical average powers, optimum processing of signals in the receiver-indicator channel and identical time of target irradiation. It should be noted that this conclusion is a direct result of the theory of detection.

The given reasoning permits estimating the suppression ratio by noise interferences of broad-band radar with coding. According to the earlier given determination, the suppression ratio means the minimum necessary ratio of the power of interference to the power of the signal within the passband of the linear part of the receiving device, in this case the optimum filter. The power of the signal at the input of the radar with coding will be determined by the power of the nonconvolute pulse of the amplitude $u_{\text{к}}$. Inasmuch as the power of the nonconvolute (long) pulse is N times less than the power of the convolute (short) pulse, then in virtue of earlier set energy equivalence the suppression ratio of the broad-band radar with coding will be N times more than the standard pulse radar with the same average power and with the same resolving power. In practice in virtue of the nonideality of the operation of the convolution of the pulse the suppression ratio will be in all (0.6-0.8)N times more than the suppression ratio of the corresponding pulse radar.

The necessary power potential of the noise jammer will remain constant if the average power of the radar with coding is not changed and the spectral density of the noise is constant. Actually, the ratio of the power of interference to the power of the signal at the input of the radar with coding is equal to

$$k_{\text{к}}=\frac{P_{\text{п}}G_{\text{п}}}{P_{\text{с к}}G_{\text{с}}}\frac{4\pi}{\sigma_{\text{ц}}}D^{2}\frac{\Delta f_{\text{пр}}}{\Delta F_{\text{п}}}\gamma_{\text{п}},$$

where $P_{\text{с к}}$ – power in the pulse radar with coding.

Since

$$P_{\text{с к}}=\frac{P_{\text{с и}}}{N},$$

where $P_{\text{с и}}$ – power of the equivalent pulse radar, then $k_{\text{к}}$ will be N times more than the ratio of the power of interference to the power of the signal at the input of the equivalent pulse radar:

$$k_{\text{и}}=\frac{P_{\text{п}}G_{\text{п}}}{P_{\text{с и}}G_{\text{с}}}\frac{4\pi}{\sigma_{\text{ц}}}D^{2}\frac{\Delta f_{\text{пр и}}}{\Delta F_{\text{п}}}\gamma_{\text{п}}.$$

However, in view of the suppression ratio of the standard pulse radar smaller than N times, the necessary power potential of the jamming transmitter $P_{\text{п}}\dot{G}_{\text{п}}$, other things being equal, in both cases will remain constant.

In the case when pulse powers of the radar with coding and the standard pulse radar with the same resolving power are equal, the power potential of the jamming transmitter necessary for suppression of the radar with coding is increased N times (or more accurately (0.6-0.8)N times).

2.5. Model Block Diagrams of Noise Interference Stations

A block diagram of a noise interference station to pulse radar is depicted in Fig. 2.14.

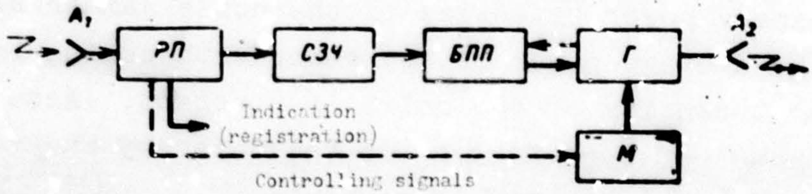

Fig. 2.14. Block diagram of the station of noise interferences to pulse radar.

Signals of the suppressed radar are taken by antenna A_1, amplified in the reconnaissance receiver RP and enter into the frequency memorization circuit SZCh where at a defined time t_3 the carrier frequency of the suppressed radar is stored. The frequency memorization circuit controls the tuning unit of the jamming transmitter BPP, with the help of which the transmitter itself is directly tuned (oscillator Г) to the carrier frequency of the suppressed radar.

From the output of the reconnaissance receiver signals join in the circuit of indication and registration, serving for their analysis and determination of the form of modulation. The latter is realized by the modulator M.

The radiation of the interference signal is carried out by the transmitting antenna A_2.

<u>Receiving and transmitting antennas</u> (A_1 and A_2) in stations of radar reconnaissance and interferences can have circular polarization, in virtue of which the coefficient γ_n in formulas of antiradar is equal to

$$\gamma_n = 0{,}5.$$

The band range of antennas is estimated by bandwidth of transmission referred to the average carrier frequency.

<u>The reconnaissance receiver</u> (RP) serves for the amplification of the received signals. Depending upon the assignment of the station of interferences it is realized either by a circuit of direct amplification or by a superheterodyne circuit.

The circuit of frequency memorization (SZCh) memorizes the carrier frequency of the suppressed radar at the assigned time. The SZCh has high requirements with respect to the recording of the carrier frequency of short-term pulse signals and its memorization for a sufficiently long time.

The tuning unit of the jamming transmitter (BPP) determines the accuracy and time of fine adjustment of the jamming transmitter at value of the carrier frequency assigned by the memory circuit. In certain stations of interferences the tuning unit in general, is absent.

Depending upon the width of the spectrum of interference signals three forms of stations of noise interferences are distinguished:

- stations of spot noise interferences;
- stations of spot-barrage noise interferences;
- stations of barrage noise interferences.

For stations of spot noise interferences the width of the radiated spectrum of interferences is selected approximately equal to the passband of the linear part of the receiver of the suppressed radar, i.e.,

$$\Delta F_{п} \approx \Delta f_{пр}.$$

For stations of spot-barrage noise interferences

$$\Delta F_{п} > \Delta f_{пр}.$$

For stations of barrage noise interferences

$$\Delta F_{п} > \Delta f_{пр}.$$

The accuracy of the tuning of the jamming transmitter in the first case is determined by the passband of the receiver of suppressed pulse radar and should be quite high. In the second case the accuracy of tuning has less stringent requirements. For the tuning of the transmitter of barrage jammings, in practice it is necessary to know the range of operation of the suppressed radar.

Oscillator of interferences (Г) depending upon the range of waves, can be carried out with electron tubes, magnetrons, traveling-wave tubes (twt) or carcinotrons (bwt).

Oscillating or amplifying tubes must provide:

- operation in a wide range of waves without considerable change in power and range efficiency;
- rapid reconstruction in frequency in the working range of waves of suppressed means;
- high power indices.

The twt and bwt possess a wide band range but have low efficiency; magnetrons have considerably high efficiency, and the highest efficiency for a variety of magnetrons is the barratron. This tube was specially created as a powerful and broad-band noise oscillator. With considerable width of the spectrum of frequencies the power of oscillations generated by the barratron is on the order higher than that of the magnetron [28].

Modulator (M) includes the source of the noise voltage and amplifying-limiting devices. In certain cases the source of the noises constitutes an independent device and does not directly appear in the modulator unit.

The source of noises can be a thyratron in a magnetic field or a noise diode of direct incandescence (saturated diode). The thyratron gives a sufficiently dense noise spectrum, but the width of spectrum is comparatively small (several megacycles per second). With help of a noise diode it is possible to obtain a noise voltage with a quite wide and uniform spectrum (ten and even hundreds of megacycles per second), but of comparatively low intensity, which does not make it possible to carryout direct modulation of the transmitter. Therefore, it is required to apply broad-band high-gain amplifiers of primary noise voltage [27].

Noises are further limited for the purpose of providing high radiation power on lateral components of the spectrum of the jamming

transmitter owing to the increase in effective percentage modulation of the transmitter by all components of the spectrum of modulating noise.

Three forms of modulation are applied: amplitude, phase and frequency. In most cases combined modulation, amplitude-frequency and amplitude-phase is observed.

2.6. Peculiarities of the Modulation of Noise Jammers

Amplitude Modulation

Amplitude noise modulation (Fig. 2.15) is applied in magnetron transmitters and triode oscillators.

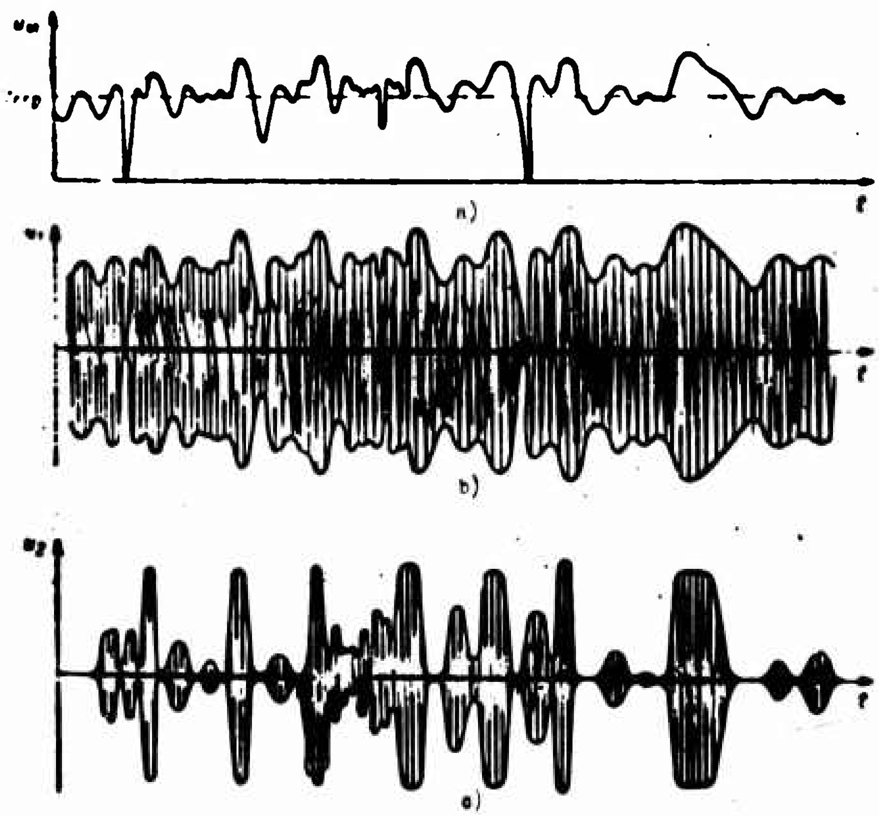

Fig. 2.15. Interference high-frequency signal amplitude modulated by noises: a) modulating voltage; b) modulation by unlimited noises; c) modulation by limited noises.

As was shown above, the noise voltage is limited. The application of limited noises provides great percentage modulation of the noises.

Figure 2.15b shows a high-frequency oscillation modulated in amplitude by practically unlimited noises. The conditions of modulation are selected so that 100% modulation was ensured by peak values (maximum overshoots) of the voltage of the modulating noise. As can be seen from the figure, the mean value of the modulation factor is small. The masking properties of interference, in general, are not worsened (quality of noise on the lateral components is great, however, the level of the lateral spectral components proves to be considerably lower than the level of the spectral component corresponding to the carrier frequency. Therefore, the spectral density of interference in a considerable part of the spectrum can appear far from sufficient for reliable suppression of the radar.

An increase in effectiveness of interferences is possible by increasing the mean value of the modulation factor. This is attained by the limitation of noise (bilateral or unilateral). The corresponding limitation of noises permits, simultaneously with an increase in mean value of the modulation factor, to expand somewhat the effective region of the spectrum of noise interference, not allowing with this the appearance overmodulation.[1] In order to be able to characterize quantitatively the percentage modulation by noise, the concept of effective factor of amplitude modulation by noise $m_{\text{эф}}$. is introduced.

Effective modulation factor by noise means the ratio of the effective value of the voltage of modulating noises $u_{\text{эф}}$ to voltage detérmining the level of limitation $u_{\text{огр}}$, i.e.,

[1]Effective region means the region of the spectrum within which the spectral density of the noise interference signal proves to be not less than the certain rated value.

where

$$m_{эф}=\frac{u_{эф}}{u_{огр}},$$

$$u_{эф}=\sqrt{P_{ш}};$$

$P_{ш}$ – power of the noise.

As an example Fig. 2.16b shows the spectrum of amplitude-modulated noise interference for the case of modulation bilaterally by limited noise voltage, which up to the limitation had a rectangular spectrum (Fig. 2.16a) [104]. Figure 2.16b gives spectral densities for a different effective modulation factor $m_{эф}$. The scale along the axis of the ordinates is selected so that the total power of the radiated oscillation in all cases equals unity.

An analysis of the given dependences (Fig. 2.16b) shows that the limitation leads to a certain expansion of the spectrum outside the border $\pm\Omega'_{п}$. Furthermore, the resulting spectrum within the effective region $(\pm\Omega'_{п})$ with limitation becomes nonuniform. Beyond the borders of the effective region the spectral density of the noise is considerably less than the spectral density of the main part of the spectrum. This fact indicates that certain expansion of the spectrum obtained with limitation cannot be recognized as being useful, since it is connected with components of the spectrum having low energy.

The considerable increase in power of lateral components owing to the redistribution of energy between carrying and lateral components of the spectrum is very important. With an increase in limitation (increase in $m_{эф}$) losses owing to the carrying components decrease, since the power of the lateral components is increased. The degree of the decrease in losses owing to the carrying components can be estimated with the help of the ratio of total power of lateral components $P_{б}$ to the power of the carrying $P_{н}$ [104]:

$$\frac{P_{б}}{P_{н}}=1-\frac{2m_{эф}}{\sqrt{2\pi}}e^{-\frac{1}{2m_{эф}}}+(m_{эф}^{2}-1)\Phi\left(\frac{1}{m_{эф}\sqrt{2}}\right).$$

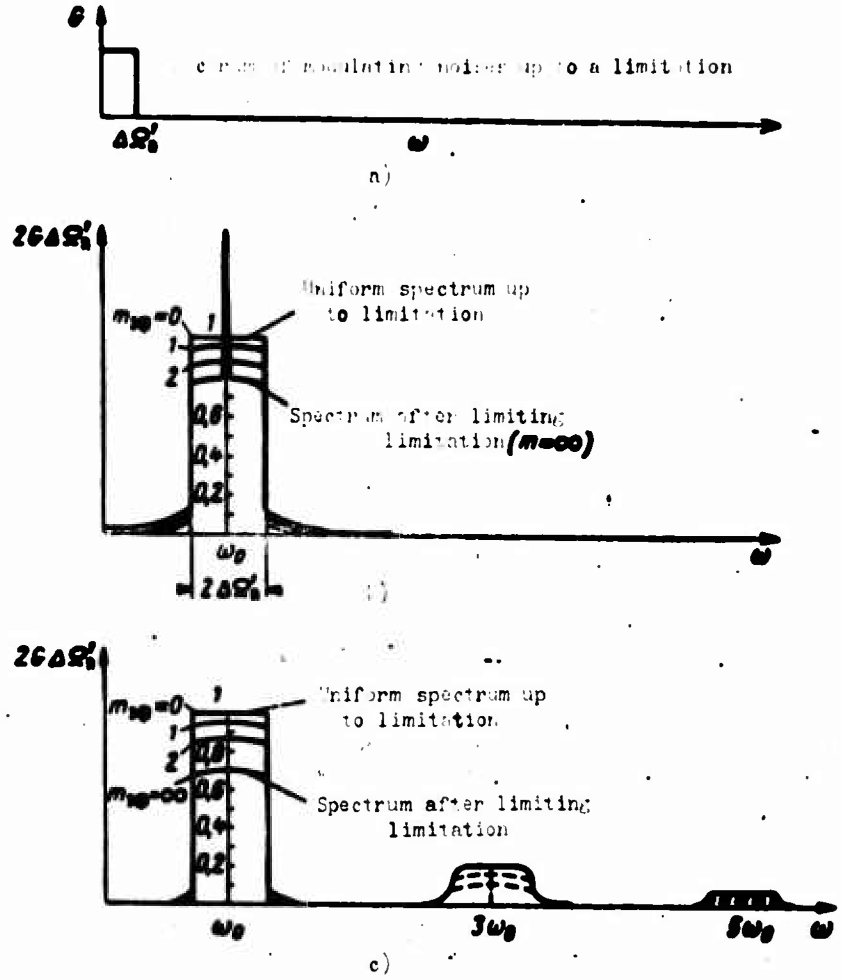

Fig. 2.16. Spectral characteristics of amplitude-modulated interference signal: a) spectrum of modulating noise; b) spectrum of limited amplitude-modulated noises; c) spectrum of limited direct noise interferences.

where θ(x) — Laplace function;

$$\theta\left(\frac{1}{m_{эф}\sqrt{2}}\right)=\frac{2}{\sqrt{\pi}}\int_{0}^{\frac{1}{m_{эф}\sqrt{2}}} e^{-t^{2}}\,dt.$$

With the help of the given expression one can be certain that the maximum power of lateral components is reached with the limiting limitation $(m_{эф}\to\infty)$ and it is equal to $P_{б\,макс}=P_{ш}$.

Thus, the limitation of modulating noises plays an important role. It permits increasing the power of the lateral components owing to the

power of the carrier component. However, the limitation involves a number of negative consequences, such as the irregularity of the spectrum, and which is the most important, the quality of the noise descends with limitation, since the noise in this case has characteristics different from characteristics of white Gaussian noise.

With amplitude-modulated noise interference the width of the radiated spectrum twice exceeds the width of the spectrum of the modulating noises:

$$\Delta\Omega_{\text{п}} = 2\Delta\Omega'_{\text{ш}}.$$

Let us note that the interference, modulated by limited noises, is not effective and therefore acceptable with any parameters of limitation. The fact is that, for example, with an increase in depth of limitation of the modulating noises (decrease in $u_{\text{огр}}$) the masking abilities of the spot noise interference worsen due to a decrease in entropy of the interference signal $H_{\text{п}}$. Figure 2.17 shows the qualitative dependences of entropy $H_{\text{п}}$ and average spectral density $\bar{G}_{\text{п}}$ of lateral components from the level of limitation of the modulating noise. With great limitations there occurs, as is sometimes said "ceiling effect" - the signal is examined against the background of too limited noises.

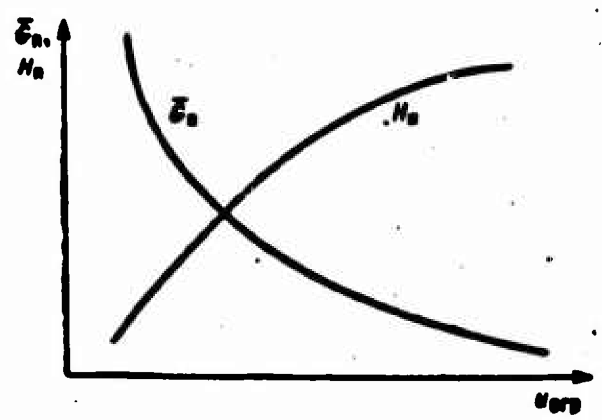

Fig. 2.17. Dependences of entropy ($H_{\text{п}}$) and average spectral density ($G_{\text{п}}$) of the noise interference signal on the level of limitation.

If $\Delta\Omega_{\text{ш}} < 2\pi\Delta f_{\text{пр}}$, then in this case the limited noise on a screen of type A has the form of nonsynchronized trapezoidal pulses (Fig. 2.18).

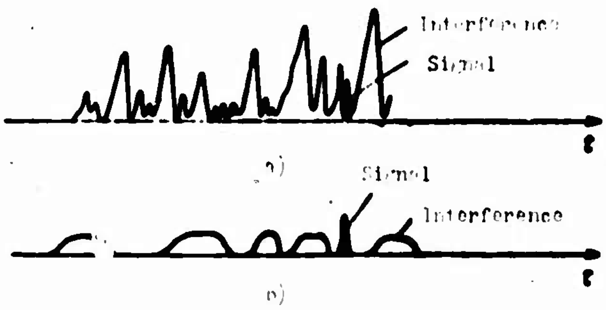

Fig. 2.18. Oscillograms of the mixture of the signal and noise: a) noises are limited normally; b) greatly limited noises.

If, however, $\Delta\Omega_{\text{ш}} > 2\pi\Delta f_{\text{пр}}$, then the amplitude-modulated noise interference, in passing through the receiving device, in spite of the amplitude limitation will be normalized [26]. At the output of the receiving channel of suppressed radar the masking properties of noise in this case are restored. So that normalization of limited noise will occur, it is necessary to fulfill the condition

$$\tau_{\text{кор}} < \frac{1}{\Delta f_{\text{пр}}},$$

where $\tau_{\text{кор}}$ – correlation time of limited noise.

In practice the level of limitation is selected by means of the compromise solution.

Bilateral limitation is conducted for the so-called direct noise (smooth) interference. In this case the limitation permits lowering the requirement for peak power. The spectrum obtained with limitation of direct noise interference considerably differs from the spectrum of the amplitude-modulated noise interference signal. This distinction appears, first of all, in the fact that part of the energy of the limited direct noise interference is distributed in vicinities of

harmonics of the center frequency of the initial spectrum (Fig. 2.16c).

Calculations show that with the limiting limitation ($u_{\text{огр}}=0$) the energy at harmonics $3\omega_0$, $5\omega_0$, $7\omega_0$, ... comprises, respectively, $\frac{8}{9\pi^2}$, $\frac{8}{25\pi^2}$, $\frac{8}{49\pi^2}$, ... from the total energy. At the harmonics there is lost about 19% of all the energy, but in all, including the energy of lateral components outside the limits of the effective region ($\pm\Delta\Omega_n$), - about 31% [104].

Phase Modulation

An analysis of the phase and frequency modulation by noises is given by a number of works. Some of the first ones investigated the spectrum of oscillations modulated in phase and frequency by noises (B. D. Sergiyevskiy [105], L. A. Vaynstein [106]). It is necessary to note also investigations [26, 107-109] and others.

High-frequency oscillation, modulated with respect to phase, is usually written in the following form:

$$u_\phi = U \sin[\omega_0 t + \psi_0 + \psi(t)],$$

where ω_0 - carrier frequency; ψ_0 - initial phase; $\psi(t)$ - stationary random process with zero mean value ($\bar{\psi}=0$) and with dispersion $[\Delta\psi_{\text{эф}}]^2$.

Parameters of phase modulation can be determined with the help of the modulation characteristic (Fig. 2.19), which is the dependence of the shift in phases of high-frequency oscillations from the modulating voltage (current). The steepness of the linear part of the modulation characteristic κ_1 determines the effective value of the phase $\Delta\psi_{\text{эф}}$, depending upon the effective value of the modulating noise voltage $u_{\text{эф}}$:

$$\Delta\psi_{\text{эф}} = \kappa_1 u_{\text{эф}}.$$

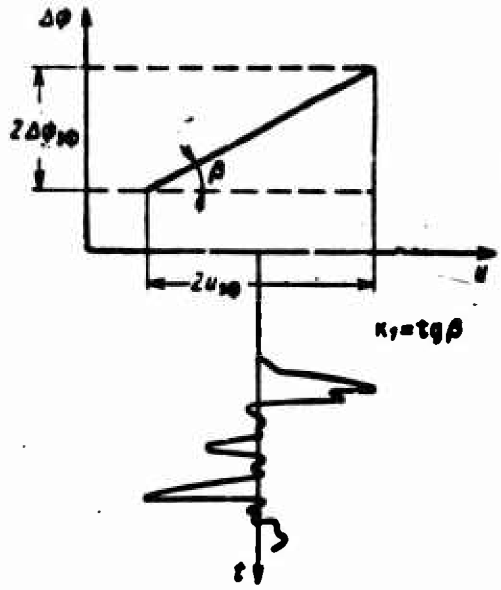

Fig. 2.19. Modulation characteristic in the case of phase modulation by noise voltage.

Depending upon the value $\Delta\psi_{эф}$ there can be two qualitatively different cases.

1) $\Delta\psi_{эф} > 1$.

In this case the width of the spectrum of oscillations modulated in phase by the noise is determined by formula [30]

$$\Delta\Omega_{\phi м} = \Delta\psi_{эф}\Omega_{п\ макс}\sqrt{\frac{2\pi}{3}},$$

where $\Omega_{п\ макс}$ – maximum frequency of the modulating spectrum.

In the particular case (Fig. 2.16), when the spectrum of modulating noise includes zero frequency:

$$\Omega_{п\ макс} = \Delta\Omega'_{ш}.$$

2) $\Delta\psi_{эф} < 1$.

The spectrum with small deviation of phase (effective value $\Delta\psi_{эф}$) consists of a discrete component on the carrier frequency and

continuous spectrum.

The form of the spectrum is similar to the spectrum obtained with amplitude modulation by noises with the effective modulation factor $m_{э\phi} < 1$. With an increase in $\Delta\psi_{э\phi}$ the intensity of the lateral components of the spectrum is increased owing to the "transfer" of energy of the carrier into the sidebands. However, with an increase in $\Delta\psi_{э\phi}$ up to the value of the order of unity the width of the spectrum is not changed. With a further increase in deviation of the phase, the width of the spectrum is increased, and the spectral density of it decreases [108, 109].

The width of the spectrum of oscillations, modulated in phase by noise (phase-modulated noise interference), is approximately equal to the doubled width of the spectrum of modulating noises. In a particular case when

$$\Omega_{в} = \Delta\Omega'_{ш},$$

we have

$$\Delta\Omega_{\phi м} = 2\Delta\Omega'_{ш}.$$

Therefore, in the fulfillment of condition $\Delta\psi_{э\phi} \ll 1$, with the help of modulation by narrow-band noise it is possible to obtain also a narrow-band noise interference signal. The form of the continuous high-frequency spectrum is similar with this to the form of the spectrum of modulating voltage.

Frequency Modulation

As is known, the instantaneous value of the frequency modulated oscillation can be written in the form

$$u(t) = U \sin\left[\omega_0 t + \int_0^t \Delta\omega(t)\,dt\right],$$

where $\Delta\omega(t)$ – random modulating function.

The form of the spectrum of oscillation, modulated with respect to frequency by noise, is completely determined by:

- the spectrum of frequencies of the modulating function;
- the effective index of modulation $m_{чм}$:

$$m_{чм}=\frac{\Delta\omega_{эф}}{\Delta\Omega_{п\,макс}},$$

where $\Delta\omega_{эф}=\overline{\Delta\omega^2(t)}$ – effective value of deviation of frequencies with noise modulation; $\Delta\Omega_{п\,макс}$ – maximum value of frequency of the modulating spectrum.

In the analysis of frequency modulated noise interference two cases are of interest.

1. The case when the effective index of modulation is large

$$m_{чм} \gg 1.$$

The width of the spectrum of the resultant radiated interference signal will be equal to

$$\Omega_{пчм} \approx \sqrt{2\pi}\, m_{чм} \Delta\Omega'_{ш},$$

i.e., the width of the spectrum of the interference proves to be larger than the width of the spectrum of modulating noise $\Delta\Omega'_{ш}$.

The masking properties of interference with a large index of modulation are relatively low in virtue of the small entropy power of interference, which is explained by the high value deviation of the modulated frequency and comparatively small values of the modulating frequency. For this reason at the output of the linear part of the receiver with action on its input of the frequency modulated interferences, the voltage can constitute a sequence of pulses, the amplitudes of which are approximately identical, and the repetition is accidental (Fig. 2.20).[1] On the screen of the indicator of type

[1]The phenomenon is observed which, to a certain degree, is similar to that one which takes place with feed to the input of the amplifier of signals from the sweep generator or frequency response characteristic meter (IChKh).

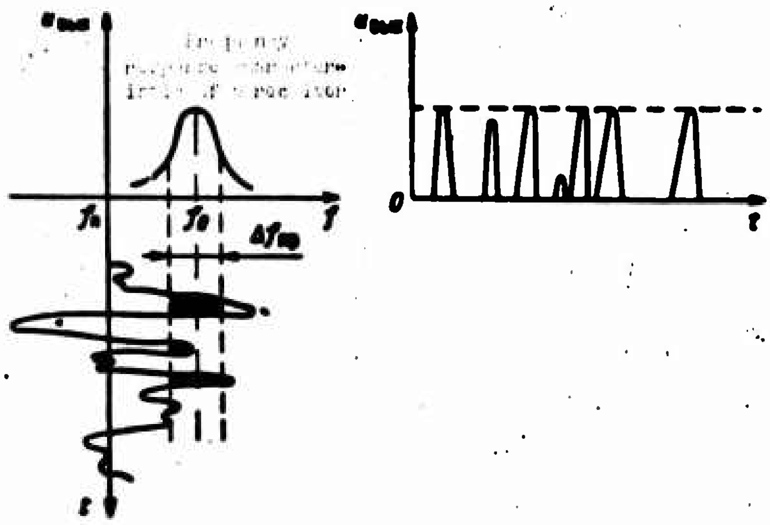

Fig. 2.20. Action of frequency modulated noise interferences on a selective receiving device.

A pulses of interferences will shift, creating a disturbing background, however, the intensity of the interference background is comparatively low. The signal, as a rule, is confidently observed on this background (Fig. 2.21). There appears the so-called "ceiling effect," which is similar to that which takes place with amplitude modulation by limited noise.

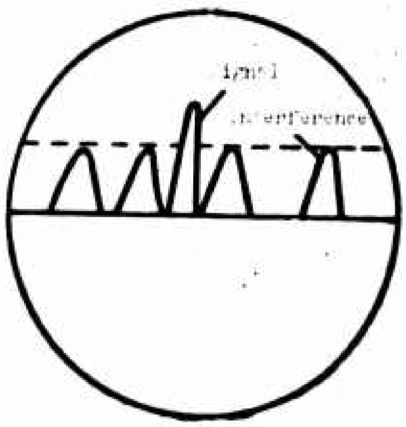

Fig. 2.21. Form of the screen with linear scan (type A) with action on the radar of frequency modulated noise interference with a large index of modulation.

The "ceiling effect" can be eliminated if the time of the frequency shift of interference within the whole range of the jamming transmitter has the order $<1/\Delta f_{пр}$. With this excitation of the circuit of the receiver is produced in random moments of time and quite often so that the output voltage will be close to the Gaussian noise.

The intensity of fluctuations at the output of the receiver of the amplitude modulated signals is maximum at the equality of the effective value of the deviation of frequency to the passband of the receiver [110].

The examined case of noise frequency modulation ($m_{чм} \gg 1$) is characteristic for the creation of spot-barrage jammings.

2. The case when $m_{чм} < 1$ $(m_{чм} = 0{,}1 \div 0{,}5)$.

The width of the spectrum of frequency modulated noise interference is equal to

$$\Omega_{н\,чм} = m^2_{чм} \Delta\Omega'_{в} \frac{\pi^2}{2}$$

Since $m^2_{чм} < 1$, then radiated interference has a quite narrow spectrum.

The "ceiling effect" will be absent if the spectrum of the modulating oscillation is selected quite wide.

In practice characteristics of the radiated interference coincide with characteristics of the modulating noise, and therefore the masking property (quality) of the frequency modulated noise interferences in this case is high.

A deficiency of such a form of interference is the small width of the radiated spectrum of frequencies, which requires accurate tuning of the jamming transmitter to the frequency of the suppressed radar.

This form of noise modulation is characteristic for the creation of spot jamming with respect to the carrier frequency.

2.7. Pulse Interferences to Radars Operating in Scan Conditions

Pulse interferences, which create false blips on the radar screen, can provide the masking of signals reflected from real targets. Furthermore, the organized pulse interferences can, to a considerable

degree, disorient the enemy and to force him to disperse his forces.

Figure 2.9d shows the screen of a panoramic radar with the action of pulse interferences on it. Together with marks from real targets there are false marks generated by active pulse interferences.

It is possible to note three forms of pulse active interferences of radar operating in scan conditions:

- multiple synchronous pulse interferences,
- simulating pulse interferences,
- random pulse interferences (KhIP).

Multiple Synchronous Pulse Interferences.

The interference constitutes a series of radio pulses radiated in answer to the accepted signal of the suppressed radar (Fig. 2.22). Radio interference pulses must correspond in form, duration and power to radio pulses of reflected signals received by the suppressed radar. Furthermore, the repetition frequency of the pulse trains should be identical with the repetition frequency of the main pulses of the suppressed radar. Otherwise the enemy is able, by applying certain methods of selection, rather simply to be liberated from the interferring signals.

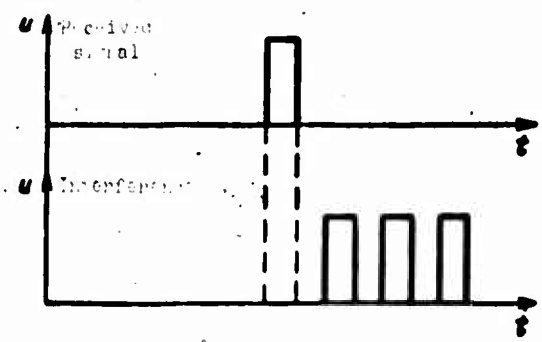

Fig. 2.22. Multiple return interference.

Figure 2.23 gives the simplest block diagram of a station of multiple return interferences constructed on the principle of the multiple relaying of radio impulses by the suppressed radar.

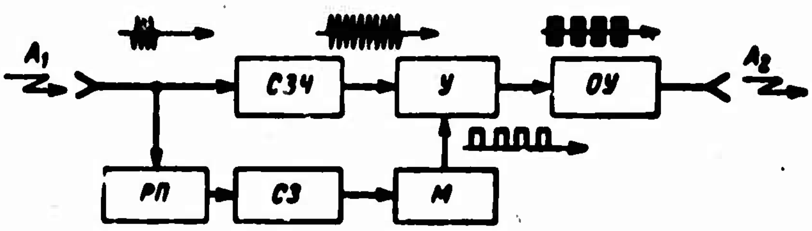

Fig. 2.23. Block diagram of a station of multiple return interferences.

The signal received by antenna A_1 proceeds to the frequency memorization circuit SZCh. Part of the power branches at the input of the reconnaissance receiver RP.

From the output of the reconnaissance receiver the signal joins the delay circuit SZ, which provides delay of the received pulse at assigned time $t_з$ (Fig. 2.24c). Modulator M forms on each of the proceeding signals a series (train) of pulses (Fig. 2.24d), with the help of which the high-frequency amplifier У is modulated.

A series of radio pulses is amplified additionally in the final amplifier OU and is radiated through antenna A_2 (Fig. 2.24e, f).

The most important link of the transmitter of synchronous pulse interferences is the frequency memorization circuit (SZCh) of the suppressed radar. This circuit memorizes the frequency of the received signal at the assigned time $t_{з.ч.}$ (Fig. 2.24b). The output voltage of the SZCh, being a harmonic oscillation with the frequency approximately equal to the carrier frequency of the radar, proceeds to the input of the amplifier У.

If the pulse repetition frequency of the suppressed radar is constant, and the memory time of the carrier frequency in the SZCh is considerably longer than the duration of the main pulse $(t_{з.ч.} \gg \tau_и)$, then it is possible to create on the radar screen false marks both lagging with respect to the target and leading the mark from the target.

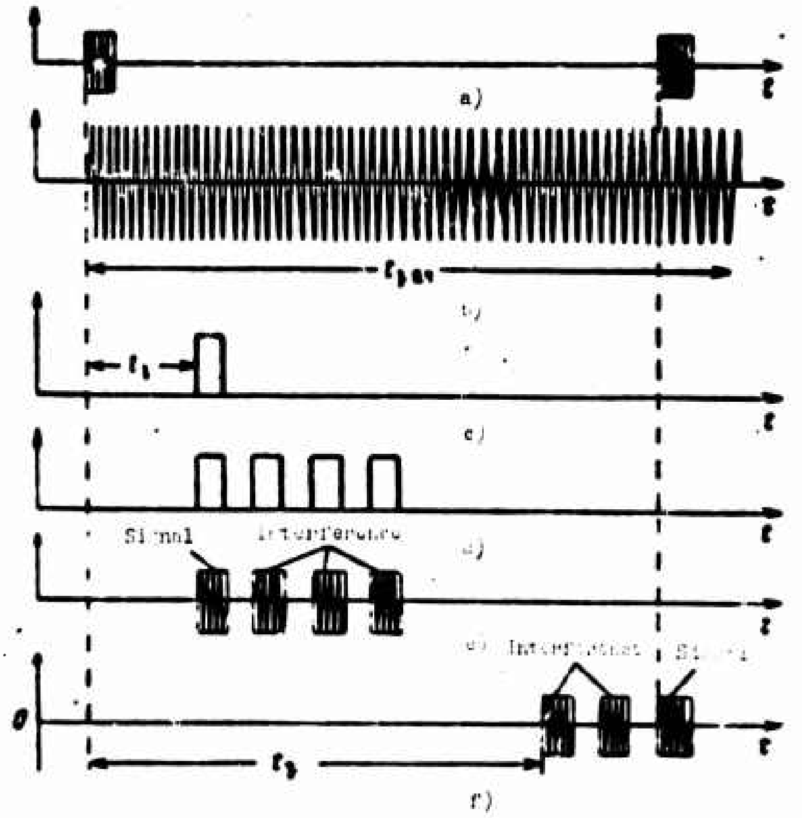

Fig. 2.24. Oscillograms of voltages at different points of the station of multiple return interferences: a) at the input of the station of interferences; b) at the output of the circuit of frequency memorization; c) at the input of the modulator; d) at the output of the modulator; e) interference pulses delayed with respect to the target; f) interference pulses leading the mark from the target.

Diagrams of voltages corresponding to these cases at the output of jamming transmitter and a representation on the screen of a plan position indicator are shown in Fig. 2.24e, f and 2.25a, b. If the power of the jamming transmitter is sufficient for suppression of the radar on lateral lobes of the antenna radiation pattern, then the false marks can be observed in directions corresponding to the lateral lobes.

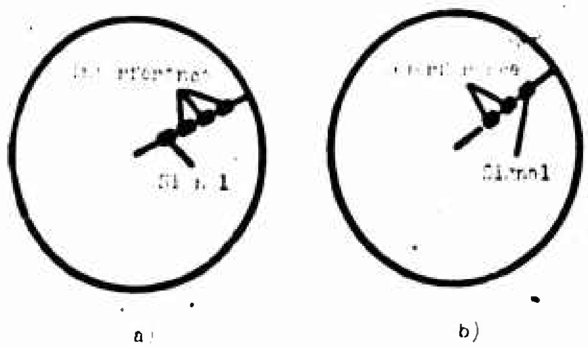

Fig. 2.25. Multiple return interferences on the screen of a plan position indicator in the case of the simulation of lagging (a) and leading (b) targets.

Simulation Pulse Interferences

Interference constitutes one (or several) radio pulses radiated in answer to the received signal of the suppressed radar with a certain delay with respect to this signal. Delay time t_3 usually changes in order to create on the screen of the suppressed radar an imitation of an actually moving target. The speed of change of delay dt_3/dt corresponds to the speed of flight of the simulated target; the delay time is selected comparatively long.

With quite high power of the jamming transmitter owing to the influence through lateral lobes on the screen of the suppressed radar, several false marks are created which move at a definite speed, which considerably complicates the work of the operators and can lead to erroneous actions of responsible persons of the antiaircraft defense system of the enemy.

The station of simulation interferences is constructed according to the same block diagram as that of the station of multiple synchronous pulse interferences (Fig. 2.23) with the only difference being that the delay in this case changes with a definite speed.

Random Pulse Interferences (KhIP)

Interference constitutes a sequence of radio pulses the basic parameters of which (repetition frequency, duration and amplitude) are changed according to the random law.

The simplest method of obtaining random pulse interferences is control by operation of a pulse modulator with the help of voltage of fluctuating noises. A block diagram of a transmitter of chaotic pulse interferences, constructed on the indicated principle, is represented in Fig. 2.26.

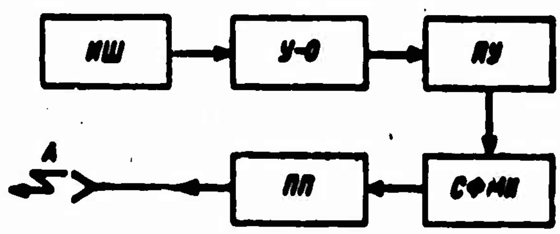

Fig. 2.26. Block diagram of a transmitter of random pulse interferences.

Noise voltage, produced by a source of noise ISh (thyratron, noise diode), is amplified and limited in a broad-band amplifier-limiter U-O at the output of which there will be formed a sequence of pulses with a constant amplitude and random repetition interval and duration. These pulses start a driven trigger circuit, which consists of a threshold device PU and circuit of the formation of modulating pulses (SFMI). The modulating pulses join the jamming transmitter. As a result radio pulses with a random duration and repetition interval will be formed.

Random pulse interferences are similar in their properties to interferences obtained as a result of amplitude modulation of the carrier by limited noise.

If the average repetition interval of KhIP

$$T_{cp} > \frac{1}{\Delta f_{np}}$$

then at the output of the suppressed receiving device there will be observed "blurred" pulses, and the effectiveness of the interference decreases for the same reason, as in the case of frequency modulated ChM noise interferences with a large index of modulation ("ceiling effect").

If $T_{cp} < \frac{1}{\Delta f_{np}}$, then at the output of the linear part of the suppressed receiver normalization of the process can take place.

CHAPTER 3

ACTIVE JAMMING OF AUTOMATIC AZIMUTH TRACKING SYSTEMS WITH SEQUENTIAL COMPARISON OF SIGNALS

3.1. Introduction

The automatic azimuth tracking (ASN) channel is the basic channel in any guidance or homing guidance circuit. With the help of the ASN channel data are obtained on angular target coordinates and their derivatives. These data are used directly in systems for the automatic tracking of selected targets.

Determination of angular coordinates of targets by radar methods is reduced in practice to measurement of angle of arrival of radio waves reflected from the targets. Usually for this the principle of the equisignal zone is widely used. Also applied are methods of direction finding in accordance with peak signal (with linear scanning of antenna beam).

At present there are two types of automatic azimuth tracking systems: systems with simultaneous and sequential comparison of signals.[1]

In ASN systems with simultaneous comparison of signals (frequently called monopulse systems or systems with instantaneous equisignal zone) determination of angular target coordinates is made in accordance

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[1]The ASN system is sometimes called a goniometric coordinator.

with results of comparison of signal parameters (amplitude, phase, frequency) picked up simultaneously by two spaced antennas. Information on angular target position in such systems, in principle, can be issued simultaneously with arrival of signal at receiving antennas, i.e., in ASN systems with simultaneous comparison of signals every received pulse carries information on target position.

In ASN systems with sequential comparison of signals (frequently called conical-scanning or integral-equisignal-zone systems) reception of signals from target at each given moment of time is handled by one antenna, whose radiation pattern rocks periodically about a certain axis (most frequently that of the equisignal zone). The coordinate is determined on the basis of comparison of the envelope of accepted signals with a certain reference signal. For this reason in order to obtain results a certain finite amount of time is required, at least equal to several pulse repetition periods or commensurable with the scanning period of the antenna.

The ASN systems with sequential comparison of signals can be actively jammed both from the target aircraft (carrying a jamming transmitter), and from a neighboring aircraft or group of aircraft (remote source of interference). Subsequently we will call interference of the first type that created from one point and that of the second type interference created from two points. Such division is necessitated by the peculiarities of action of interference radiated by sources carried by and apart from the covered aircraft.

In this chapter is studied interference created from one point. Such interference includes signals with different conditions of low-frequency amplitude modulation. Interference created from two points is examined in Chapter 4. The method of their consideration, and in many cases results obtained in Chapter 4, also can be extended to ASN systems with sequential comparison of signals.

The effectiveness of organized interference in considerable degree is determined by the form of the direction finding characteristic of the suppressed ASN system. Therefore we will briefly examine real direction finding characteristics by which is understood

the dependence of the output voltage of the phase detector on angular displacement of target relative to the equisignal direction.

3.2. Direction Finding Characteristics of ASN Systems with Sequential Comparison of Signals

A simplified block diagram of an ASN system with sequential comparison of signals is in Fig. 3.1. For simplicity direction finding in one plane is examined.

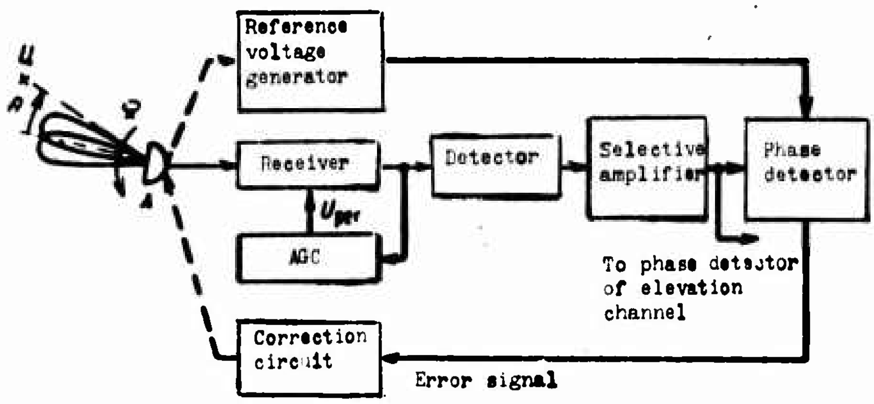

Fig. 3.1. Block diagram of ASN system with sequential comparison of signals.

Receiving antenna A scans in space with angular frequency Ω. In Fig. 3.2a is depicted the radiation pattern of the scanning antenna. The axis of the equisignal zone is displaced with respect to the peak of the pattern by angle θ_0.

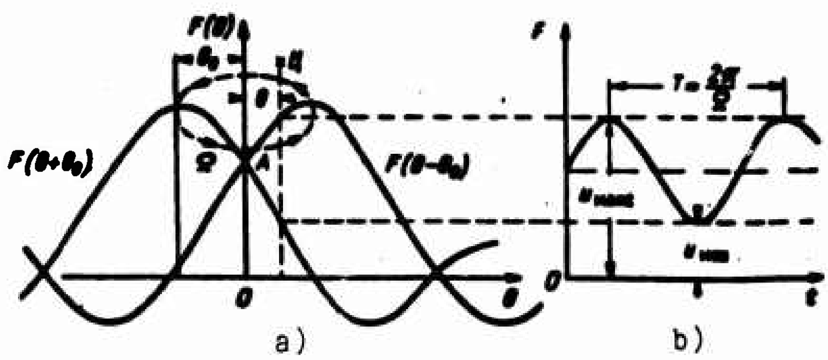

Fig. 3.2. Diagram of formation of error signal: a) cross section of radiation pattern of scanning antenna; b) formation of error signal.

If the antenna radiation pattern is symmetric, the trace of the point corresponding to the peak of the pattern makes a circle in the picture plane (Fig. 3.3).

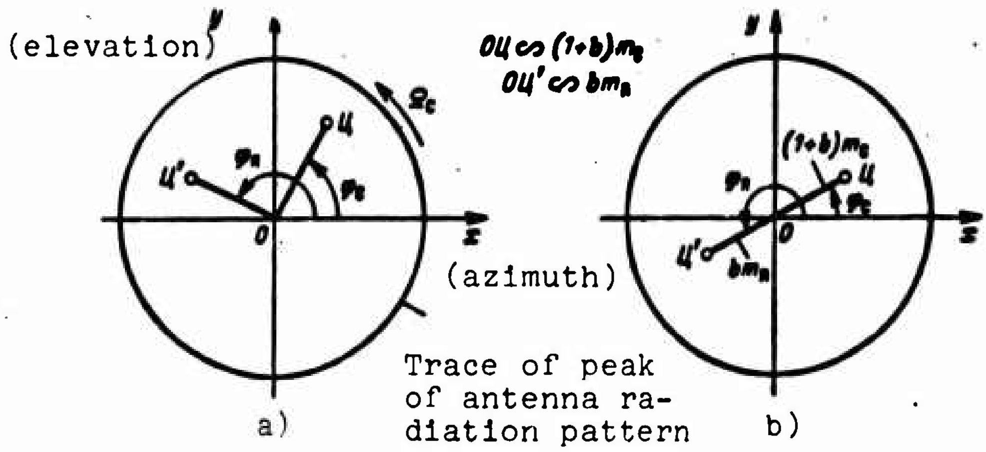

Fig. 3.3. Trace of peak of antenna radiation pattern in picture plane.

In the presence of an error in tracking the envelope of the receiver input signal is a periodic curve, close to a sinusoid (Fig. 3.2b). The phase of the envelope is determined by angle ϕ_c, depending on the position of the target relative to a certain axis in the picture plane, while its amplitude is determined by the displacement angle θ.

In first approximation the signal at the receiver input can be recorded in the form of an amplitude-modulated signal, the modulation factor and amplitude of which depend on the value of the tracking error θ:

$$u=U_0U(\theta)[1+m(\theta)\cos(\Omega t+\varphi_c)]\cos\omega t, \tag{3.1}$$

where U_0 – amplitude of signal;

$$U(\theta)=\frac{u_{макс}+u_{мин}}{2}=\frac{|F(\theta_0-\theta)|+|F(\theta_0+\theta)|}{2}; \tag{3.2}$$

$$m(\theta)=\frac{u_{макс}-u_{мин}}{u_{макс}+u_{мин}}=\frac{|F(\theta_0-\theta)|-|F(\theta_0+\theta)|}{|F(\theta_0-\theta)|+|F(\theta_0+\theta)|}; \tag{3.3}$$

$F(\theta)$ – standardized antenna radiation pattern.

Representation of signal in form (3.1) gives small error (10-15%) up to values of displacement angle of $\theta \approx \theta_{0,5}$. For greater values of displacement angle θ approximation of input signal by a function of form (3.1) gives considerable error – of the order of 50%. Formulas (3.1)-(3.3) are valid under the assumption of ideality of AGC.

Let us evaluate the influence of AGC on the direction finding characteristic of radar with conical scanning.

At the output of a receiver with regulated gain $K(U_p)$ we have

$$u_{вых} = K(U_p) U_0 U(\theta)[1 + m(\theta)\cos(\Omega t + \varphi_c)]\cos\omega_{пр} t, \qquad (3.4)$$

where $\omega_{пр}$ – intermediate frequency.

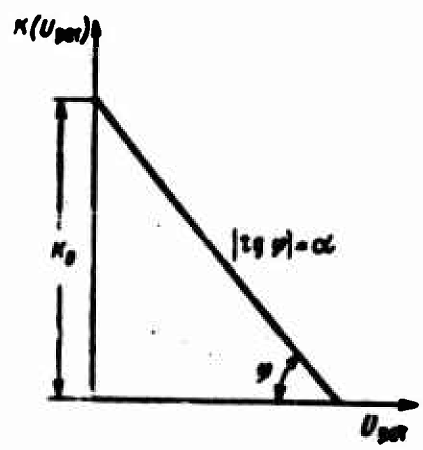

Fig. 3.4. Regulating characteristc of AGC.

The receiver gain is regulated by the AGC in dependence upon intensity of input signal. Let us take the regulating characteristic of the AGC (i.e., dependence of receiver gain K on grid bias voltage U_p of the controlled tubes) as linear (Fig. 3.4)

$$K = K_0 - \alpha U_{рег}, \qquad (3.5)$$

where

$$\alpha = |tg\,\varphi|.$$

We will designate the amplitude of the voltage at the AGC detector input by U_c and the transfer constant of AGC detector and amplifier by K_2. Then in the absence of delay voltage

$$U_{рег} = K_2 U_c. \qquad (3.6)$$

Voltage U_c under steady-state conditions is related to the envelope of the signal at the receiver input $U_{c\,вх}$ as

$$U_c = KU_{c\,вх}, \tag{3.7}$$

where

$$U_{c\,вх} = U_0 U(\theta) = U_0\frac{|F(\theta_0-\theta)|+|F(\theta_0+\theta)|}{2}.$$

Solving system of equations (3.5), (3.6), and (3.7) for K, we obtain an expression for receiver gain

$$K = \frac{K_0}{1+\alpha\kappa_2 U_{c\,вх}} = \frac{K_0}{1+\mu[|F(\theta_0-\theta)|+|F(\theta_0+\theta)|]}, \tag{3.8}$$

where

$$\mu = \frac{\alpha\kappa_2 U_0}{2}.$$

Passing the signal through the square-law detector, we obtain at the output of the selective amplifier, tuned to the scan frequency,

$$u_{c\,y} = K^2\kappa_{c\,y}U^2_{c\,вх}m(\theta)\cos(\Omega t+\varphi_c), \tag{3.9}$$

where $\kappa_{c\,y}$ – transfer constant of detector and selective amplifier.

Assuming that the phase detector multiplies the input signal $u_{c\,y}$ by the sinusoidal reference signal $u_{оп}(t) = u_{оп}\cos\Omega t$ with subsequent averaging of obtained product, finally from (3.9), taking into account (3.8), we find the expression for the direction finding characteristic of the ASN system with sequential comparison of signals (with square-law detection)

$$u_{ф\,д} = \overline{u_{c\,y}u_{оп}} = \kappa_{ф\,д}\frac{\mu^2[F^2(\theta_0-\theta)-F^2(\theta_0+\theta)]}{\{1+\mu[|F(\theta_0-\theta)|+|F(\theta_0+\theta)|]\}^2}, \tag{3.10}$$

where

$$\kappa_{ф\,д} = \frac{\kappa_{c\,y}\kappa' K_0^2}{2\alpha^2\kappa_2^2},$$

κ' – transfer constant of phase detector.

Analogously, if the detector is linear, we obtain

$$u'_{ф\,д} = \kappa'_{ф\,д}\frac{\mu[|F(\theta_0-\theta)|-|F(\theta_0+\theta)|]}{1+\mu[|F(\theta_0-\theta)|+|F(\theta_0+\theta)|]}. \tag{3.10a}$$

where

$$\varkappa'_{\phi\,\text{д}} = \frac{\varkappa_{c\,y}\varkappa' K_0}{2\pi\varkappa_0}.$$

For greater amplitudes of input signals ($\mu \to \infty$) the direction finding characteristics (3.10) and (3.10a) becomes idealized:

$$u_{\phi\,\text{д}} = \varkappa_{\phi\,\text{д}} \frac{|F(\theta_0-\theta)| - |F(\theta_0+\theta)|}{|F(\theta_0-\theta)| + |F(\theta_0+\theta)|}.$$

In Figs. 3.5 and 3.6 are represented direction finding characteristics of the ASN system with sequential comparison of signals, plotted in accordance with formula (3.10) for $\theta_0/\theta_{0,5} =$ 0.3 and 0.5 for different values of signal intensity (μ = 1, 3, 10). Plotting was done for the case when the antenna radiation pattern is described by function

$$F(\theta) = \frac{\sin\frac{\pi D}{\lambda}\theta}{\frac{\pi D}{\lambda}\theta} = \frac{\sin x}{x},$$

where D — diameter of antenna reflector; λ — wavelength.

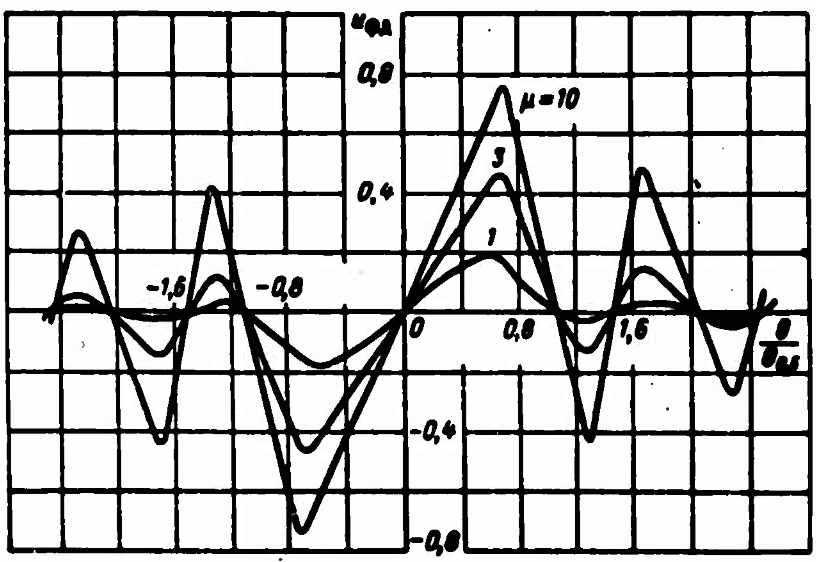

Fig. 3.5 Direction finding characteristics of ASN system with sequential comparison of signals for $\frac{\theta_0}{\theta_{0,5}} = 0.3$.

As follows from the graphs, the direction finding characteristics display, in general, nonlinear dependence of control voltage $u_{\phi\,\text{д}}$

on angular error θ. Direction finding characteristics can be considered linear only for small displacement angles $\theta/\theta_{0.5}<0.5$. Direction finding characteristics have false equisignal directions (minor lobes), which becomes stronger with growth of signal intensity.

Signal amplitude renders significant influence on the form of direction finding characteristics.

An increase of parameter μ, proportional to signal amplitude, leads to growth of angular distance between peaks of characteristics.

Let us note that the formulas obtained for direction finding characteristics (3.10) and (3.10a) and plotted graphs (Fig. 3.5) permit performing quantitative analysis of dependences only for angles $\theta\leqslant\theta_{0.5}$. For greater displacement angles these formulas can give considerable error, but the qualitative picture is not distorted.

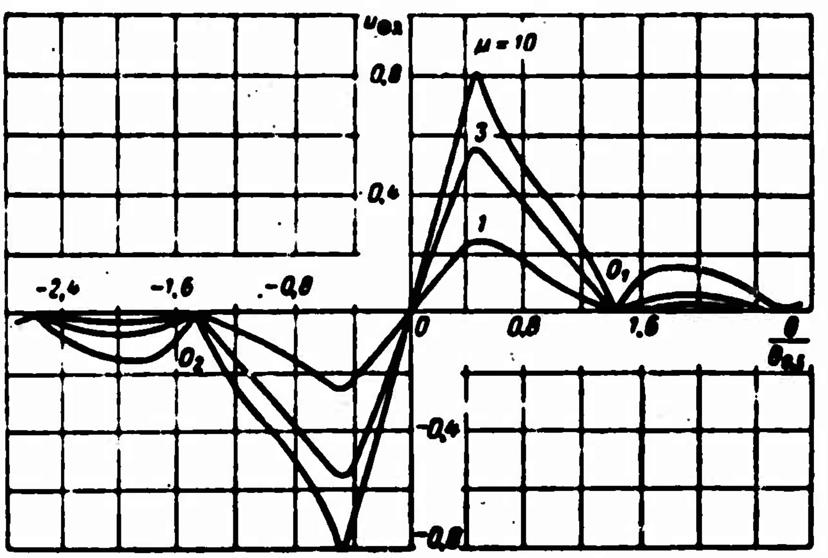

Fig. 3.6. Direction finding characteristics of ASN system with sequential comparison of signals for $\frac{\theta_0}{\theta_{0.5}}=0.5$.

3.3. Selective Jamming of Scanning Frequency

Selective jamming of scanning frequency is created by way of reradiation of signals received from the radar to be jammed with, simultaneous modulation of them in amplitude, where the scanning

frequency of this radar is known beforehand or is determined directly in the jamming process. In practice such interference can be created only for that radar whose scanning frequency is known exactly or can be determined in the jamming process (radar with unguarded scanning frequency).

The simplest block diagram of jamming station aimed at scanning frequency is shown in Fig. 3.7.

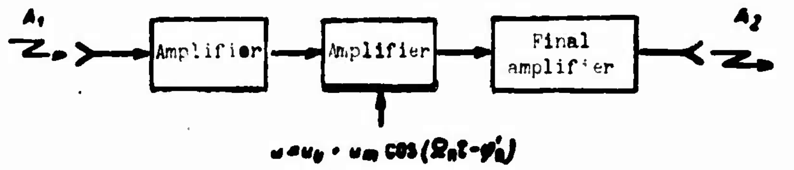

Fig. 3.7. Block diagram of transmitter for selective jamming of scanning frequency.

The received signal is strengthened and modulated in amplitude by a voltage of form

$$u = U_0 + U_m \cos(\Omega_n t - \varphi'_n),$$

after which it is radiated by transmitting antenna A_2.

Let us consider the effect on a radar with conical scanning of desired signal and interference aimed at the scanning frequency.

We will assume that scanning takes place only during reception (guarded scanning frequency, passive or semiactive conditions of direction finding) and that desired and interference signals have identical carrier frequencies and identical phases of high-frequency oscillations (phases of carrier). Furthermore, it is assumed that interference and signal are continuous. Let us note that allowing for the pulse character of signal presents no difficulties and can be performed by way of multiplication of continuous signal by the pulse function of time. We will be interested in the steady state of the tracking system during the influence of interference and desired signals. Transient conditions in this case are not examined.

Useful and interference signals at the antenna input, taking into account the assumptions made, can be represented in the following way:

$$u_{c}(t)=U_{c}e^{j\omega t}, \tag{3.11}$$

$$u_{п}(t)=U_{п}[1+m_{п}\cos(\Omega_{п}t-\varphi_{п})]e^{j\omega t}. \tag{3.12}$$

Here $U_{c}=\kappa\sqrt{P_{c\,вх}}$ – amplitude of desired signal; $U_{п}=\kappa\sqrt{P_{п\,вх}}$ – amplitude of interference signal; κ – proportionality factor; $m_{п}$ – modulation factor of interference signal; $\Omega_{п}$, $\varphi_{п}$ – angular frequency and phase of modulating voltage.

The scanning antenna carries out amplitude modulation of the mixture of signal and interference, as a result of which the voltage at the receiver input has the form:

$$u_{A}=[u_{c}(t)+u_{п}(t)][1+m_{c}\cos(\Omega_{c}t-\varphi_{c})], \tag{3.13}$$

where m_c is the modulation factor caused by scanning of receiving antenna, the value of which is proportional to the displacement angle between the equisignal line and direction to target θ (in this case the jammer);[1] Ω_c – angular frequency of scanning; ϕ_c – initial phase, determined by position of target relative to a certain axis in plane perpendicular to equisignal line (Fig. 3.3a).

At the receiver output (detector input) from (3.13), taking into account (3.12), we obtain

$$u_{пр.м}=\kappa_{1}U_{c}[1+b+bm_{п}\cos(\Omega_{п}t-\varphi_{п})]\times$$
$$\times[1+m_{c}\cos(\Omega_{c}t-\varphi_{c})]e^{j\omega_{пр}t}, \tag{3.14}$$

where

$$b=\frac{U_{п}}{U_{c}}=\sqrt{\frac{P_{п\,вх}}{P_{c\,вх}}}; \tag{3.15}$$

$\omega_{пр}$ – intermediate frequency.

---

[1]During the tracking of one target, not creating interference, the modulation factor is close to zero (in steady-state operating conditions).

If the detector is linear, at its output is separated envelope

$$u_{д}=\kappa_1\kappa_{д}U_c\,[(1+b)+(1+b)m_c\cos(\Omega_c t-\varphi_c)+ \\ +bm_п\cos(\Omega_п t-\varphi_п)+bm_п m_c\cos(\Omega_п t-\varphi_п)\cos(\Omega_c t-\varphi_c)]. \quad (3.16)$$

At the output of the narrow-band selective amplifier, whose resonant frequency coincides with the scanning frequency, we will have only components of frequency Ω_c, but also of frequency $\Omega_п$, if the latter differs from Ω_c by a value no larger than the bandpass of the selective amplifier $\Delta\Omega$:

$$u_{o\,y}=\kappa_1\kappa_{д}\kappa_{c\,y}U_c\,[(1+b)m_c\cos(\Omega_c t-\varphi_c)+ \\ +bm_п\cos(\Omega_п t-\varphi_п)]. \quad (3.17)$$

As a result of multiplication of working and reference signals in the phase detector and subsequent averaging in the filter, the error signal for elevation and azimuth channels respectively has the form:

$$u_y=\kappa\,\{(1+b)m_c\sin\varphi_c+bm_п\sin[(\Omega_п-\Omega_c)t+\varphi_п]\}, \quad (3.18)$$

$$u_x=\kappa\,\{(1+b)m_c\cos\varphi_c+bm_п\cos[(\Omega_п-\Omega_c)t+\varphi_п]\}. \quad (3.19)$$

The first terms in (3.18) and (3.19) represent the useful component of the error signal induced by the displacement angle of the target (jammer) relative to the equisignal line.

The second terms in (3.18) and (3.19) are the result of the action of interference.

The effectiveness of the examined interference depends in considerable measure on the difference between modulation frequency $\Omega_п$ and scanning frequency Ω_c, i.e., on

$$\Delta\Omega=|\Omega_п-\Omega_c|.$$

Only two case of suppression of ASN systems by the examined method are of practical interest: when

$$\Omega_п=\Omega_c, \quad (3.20)$$

$$|\Omega_п-\Omega_c|<\Delta\Omega_к. \quad (3.21)$$

Here $\Delta\Omega_к$ is the bandpass of the closed-loop ASN system.

In first case from (3.18) and (3.19) we obtain

$$u_y = \kappa\,[(1+b)\,m_c \sin\varphi_c + b m_п \sin\varphi_п], \tag{3.22}$$

$$u_z = \kappa\,[(1+b)\,m_c \cos\varphi_c + b m_п \cos\varphi_п]. \tag{3.23}$$

From (3.22 and (3.23) it is clear that interference of the examined form in its effect is equivalent to a certain fictitious target Ц', not coinciding in space with true target Ц (Fig. 3.3a). In other words, the interference signal generates false information, simulating the appearance of a second, fictitious, target Ц', whose angular coordinates differ from those of true target Ц. In accordance with the functional principle of the ASN system the equisignal line in this case is automatically oriented to the peak center of mass of sources Ц and Ц', located on a straight line connecting points Ц and Ц' (Fig. 3.3b). Such behavior of the ASN system can be explained with the help of the direction finding characteristic of the radar (Fig. 3.8).

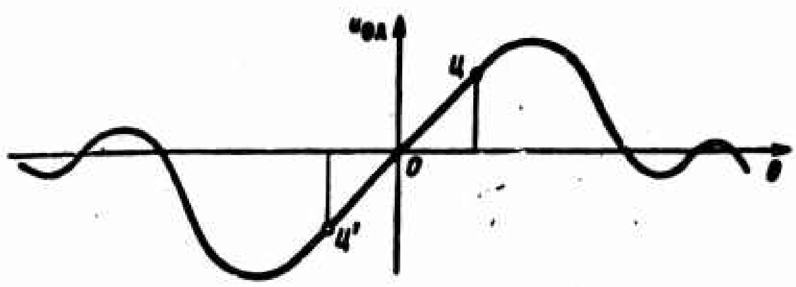

Fig. 3.8. Position of true and fictitious targets on direction finding characteristic.

Like any tracking system based on the principle of automatic adjustment for deviation of controlled quantity from a certain assigned value the ASN system functions in such a way as to always ensure equality to zero of voltage at the output of the phase detector. Error voltage at the output of the phase detector is equal to zero and the state of the system is stable when the zero of the direction finding characteristic is located on segment ЦЦ' at the point of peak "center of mass" of true and false targets and the slope of the characteristic is positive at this point. With some other position

of the zero of the direction finding characteristic the equilibrium of the system is disturbed, which can be verified directly by shifting the zero of the direction finding characteristic to various points in interval ШШ', and also beyond the limits of this interval.

The ASN system automatically shifts the antenna in such a way that the axis of the equisignal line is on segment ШШ', where phases φ_c and $\varphi_п$ in steady-state operation will differ evenly by 180°, independently of their value at the moment the jamming transmitter is switched on (Fig. 3.8). Voltages of error signal (3.22) and (3.23), in accordance with the above reasonings, will be equal to zero if

$$\varphi_c = \varphi_п \pm 180°, \tag{3.24}$$

$$(1+b)m_c = bm_п. \tag{3.25}$$

Conditions of equilibrium of the system during the influence on it of two signals (3.24) and (3.25) are called conditions of balance of phases and amplitudes respectively.

From (3.25) we obtain the formula for m_c:

$$m_c = m_п \frac{b}{1+b}. \tag{3.26}$$

Formula (3.26) establishes the relationship between modulation factors of useful and interference signals. Graph corresponding to expression (3.26) are shown in Fig. 3.9.

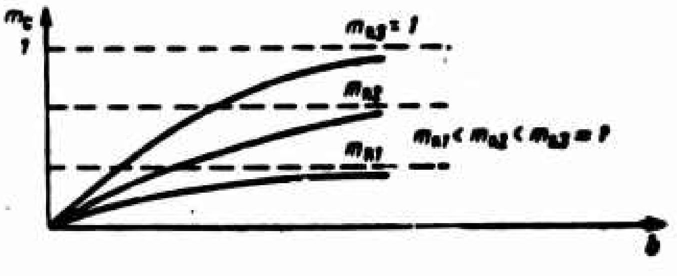

Fig. 3.9. Dependence of modulation factor of signal at receiver input on the signal-to-noise ratio of the ASN system during the action of interference on scanning frequency.

As follows from formula (3.26) and Fig. 3.9, in principle, even with infinite power of interference it is impossible to obtain a modulation factor of desired signal m_c greater than the modulation factor of the interference signal $m_{п}$. Physically this is explained by the fact that in the composition of the spectrum of the interference signal along with the two sideband components, carrying interference information, is the carrier frequency, transmitting information on the true angular coordinates of the source of interference signals.

With constant value of interference modulation factor the ratio of carrier power to the power of sideband components of interference also remains constant and does not depend on the power of the jamming transmitter. Therefore when b >> 1 the desired signal has practically no effect on angular error, and the amount of angular deflection of the antenna of the jammed radar, and consequently the value of m_c, remain constant. The interference carrier compensates for the effect of its sideband components, and equilibrium is established in the system.

If the desired signal were entirely absent and the ASN system were influenced only by the interference signal, after completion of transient conditions the equisignal line of the antenna would be deflected by a constant angle, corresponding to equality of modulation factors of useful and interference signals $(m_c = m_п)$. This angle will theoretically remain constant with increase of power of the interference signal (dotted lines in Fig. 3.9).

The amount of deflection of antenna is limited by the interference modulation factor $m_п$ and parameters of the antenna system of the jammed radar. Let us find the dependence of angle deflection on the shown parameters.

By definition of the modulation factor

$$m = \frac{U_{макс} - U_{мин}}{U_{макс} + U_{мин}}, \tag{3.27}$$

where $U_{макс}$ and $U_{мин}$ are maximum and minimum amplitudes of modulated oscillation.

In reference to the examined case the formula for m_c can be recorded in the following way:

$$m_c = \frac{F(\theta_0 - \theta) - F(\theta_0 + \theta)}{F(\theta_0 - \theta) + F(\theta_0 + \theta)}. \quad (3.28)$$

Function $F(\theta)$ can be expanded in series in the neighborhood of point A (Fig. 3.2). If this expansion is limited to only the first two members,

$$m_c = \frac{2F'(\theta_0)}{2F(\theta_0)}. \quad (3.29)$$

In accordance with the definition the modulation factor is essentially a positive quantity; therefore

$$m_c = \left|\frac{F'(\theta_0)}{F(\theta_0)}\right| \theta. \quad (3.30)$$

The factor $\left|\frac{F'(\theta_0)}{F(\theta_0)}\right|$ determines the slope of the direction finding characteristic.

Frequently the major lobe of an antenna radiation pattern is approximated by the function

$$F(\theta) = e^{-1,4\left(\frac{\theta}{\theta_{0,5}}\right)^2}, \quad (3.31)$$

where $\theta_{0,5}$ is the half-power width of the radiation pattern.

Accordingly when $\theta = \theta_0$

$$F'(\theta) = -2,8\frac{\theta_0}{\theta_{0,5}^2} F(\theta_0). \quad (3.32)$$

Substituting (3.33) in formula (3.26), we obtain the expression for angular deflection of sight line:

$$m_c = 2,8\frac{\theta_0 \theta}{\theta_{0,5}^2}. \quad (3.33)$$

From (3.30), taking into account (3.32), we obtain

$$\frac{\theta}{\theta_{0,5}} = \frac{1}{2,8}\frac{\theta_{0,5}}{\theta_0}\frac{b}{1+b} m_{\text{м}}. \quad (3.34)$$

The graph of dependence of $\theta/\theta_{0,5}$ on b for $\frac{\theta_0}{\theta_{0,5}} = 0.5$ is presented in Fig. 3.10.

Maximum value of angular deflection of sight line corresponds to $m_п = 1$ and b → ∞.

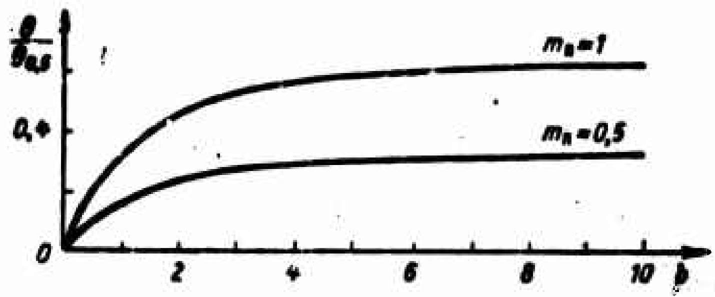

Fig. 3.10. Dependence of angular error of target tracking on signal-to-noise ratio during selective jamming of scanning frequency $\left(\frac{\theta_0}{\theta_{0,5}} = 0,5\right)$.

It can be found from equation

$$\frac{\theta_{макс}}{\theta_{0,5}} = \frac{1}{2,8}\frac{\theta_{0,5}}{\theta_0}. \qquad (3.35)$$

If it is considered that $\theta_0/\theta_{0,5} = 0,5$, then $\theta_{макс} \approx 0,7\theta_{0,5}$.

Analysis of dependence of error in angular tracking on b (Fig. 3.10) makes it possible to estimate the minimum necessary value of the ratio of power of interference signal to power of desired signal (suppression ratio $k_п$).

We will not examine the second case of selective jamming of scanning frequency, corresponding to inequality (3.21).

As follows from (3.18) and (3.19), in this case phase is a linear function of time

$$\varphi'_п(t) = (\Omega_п - \Omega_c)t + \varphi_п.$$

Since $\Omega_п - \Omega_c < \Delta\Omega_к$, and $\Delta F_к = 2\pi\Delta\Omega_к$ has a value of the order of a few Hertz, this function will be slowly changing.

The value of $\varphi'_п$, as it is known, determines the position of the fictitious target. A change of $\varphi'_п$ generates a shift of fictitious target (Ц″ in Fig. 3.11), which in turn leads to displacement of axis of equisignal zone (O' in Fig. 3.11), inasmuch as system should automatically work out the condition of phase balance

$$\varphi'_п = \varphi_c \pm 180^\circ.$$

Consequently, the axis of the equisignal zone, in deviating from direction to target by angle θ, will revolve around this direction, forming the surface of a cone.

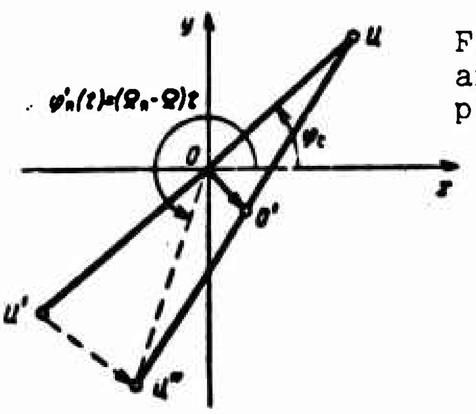

Fig. 3.11. Positions of true and fictitious targets in picture plane.

3.4. Barrage Jamming of Scanning Frequency

Selective jamming of scanning frequency is possible only in the case when, with precision determined by bandpass of suppressed coordinator (ASN system), the scanning frequency is known.

In practice during the suppression of radar with guarded scanning frequency (scanning of receiving antenna only) the value of the latter is unknown. This makes it necessary to resort to barrage jamming, making counteraction in a certain range of scanning frequencies possible.

We distinguish two methods of formation of scanning frequency barrage signals:

– amplitude modulation of carrier-by low-frequency noise with continuous or discrete spectrum, covering a certain range of possible scanning frequencies of ASN systems;

– amplitude modulation of carrier by sinusoidal voltage whose frequency is changed systematically in the range of possible scanning frequencies (sweep-through jamming).

Interference of both forms, in principle, can be created in the same manner as selective interference, i.e., with the help of several stages of amplifier-lays, in one of which is carried out modulation by voltages ξ_1 or ξ_2, depending upon which of the two shown methods is used to create interference (Fig. 3.12).

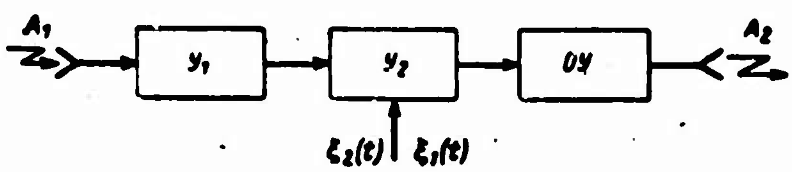

Fig. 3.12. Principle of creation of barrage jamming of scanning frequency.

For interferences of the first form modulating voltage $\xi_1(t)$ constitutes low-frequency noise, the spectrum of which covers the range of possible scanning frequencies $\Delta\Omega_п$ (Fig. 3.13). Modulating voltage $\xi_1(t)$ can also constitute a set of sinusoids whose frequencies differ by an amount equal to (or smaller than) the bandpass of the suppressed device $\Delta\Omega_к$, while their number N is selected on the basis of the condition of coverage of the range of possible scanning frequencies

$$N=\frac{\Delta\Omega_п}{\Delta\Omega_к},\quad N>1. \tag{3.36}$$

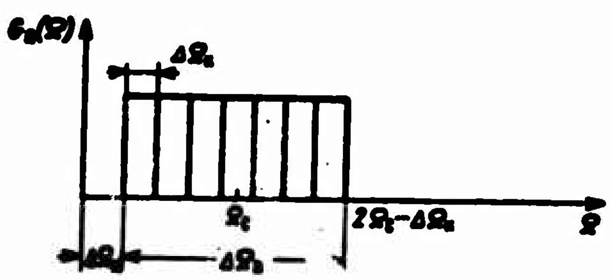

Fig. 3.13. Spectrum of envelope of scanning blanket barrage noise.

Interference of the second type can be created by periodically changing the frequency of interference modulation (ξ_2) in the range of possible scanning frequencies, for example, per sawtooth law (Fig. 3.14).

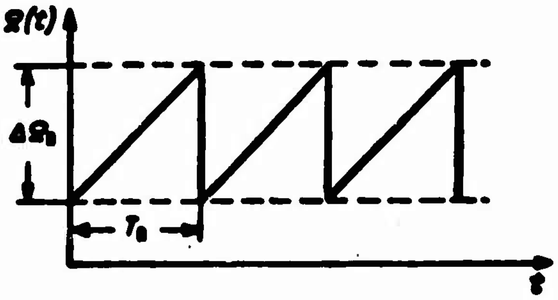

Fig. 3.14. Law of change of frequency for the case of creation of scanning frequency barrage jamming of sweep-through type.

Blanket Barrage Jamming of Scanning Frequency

Let us examine the influence of low-frequency noise on an ASN system with conical scanning.

We will consider the modulating noise steady. This will allow us to represent it in the form of a combination of sinusoidal oscillations, the amplitude U_i of which is determined by the effective value of noise:

$$\xi_1(t)=\sum_{i=1}^{N} U_i(t)\cos[\Omega_i t-\varphi_i(t)], \qquad (3.37)$$

where $U_i(t)$ and $\phi_i(t)$ are slowly changing functions of time (as compared to $\cos \Omega_i t$); N is determined by relationship (3.36).

It would be more correct in this case to consider that the modulating voltage actually constitutes the sum of harmonic oscillations of identical amplitude but different frequency, where the difference between adjacent frequencies is equal to $\Delta\Omega_K$.

The signal, modulated in amplitude simultaneously by several harmonic oscillations of various frequency Ω_i, can be represented in the following way:

$$u_{\text{п}}(t)=U_{\text{п}}\left[1+\sum_{i=1}^{N} m_{\text{п}i}\cos(\Omega_i t-\varphi_i)\right]e^{j\omega t},$$

where $U_{\text{п}}$ – amplitude of modulated oscillation; $m_{\text{п}i}$ – modulation factor provided by the i-th component of modulating noise.

At the input of the antenna of the suppressed radar we will have

$$u_{\text{вх А}} = u_{\text{п}}(t) + u_{\text{c}}(t),$$

where

$$u_{\text{c}}(t) = U_{\text{c}} e^{j\omega t};$$

U_c – amplitude of signal.

At the output of the receiver, taking into account the same assumptions made during the analysis of the influence of spot jamming, we obtain

$$U_{\text{вых пр}} = KU_{\text{c}} \left[1 + b + b \sum_{i=1}^{n} m_{\text{п}i} \cos(\Omega_i t - \varphi_i) \right] \times$$
$$\times [1 + m_{\text{c}} \cos(\Omega_{\text{c}} t - \varphi_{\text{c}})] e^{j\omega_{\text{пр}} t},$$

where

$$b = \frac{U_{\text{п}}}{U_{\text{c}}}.$$

At the output of the error signal filter (selective amplifier) will be only harmonic whose frequencies differs from the scanning frequency by not more than the bandpass of the filter.

For simplicity we will consider that the spectrum of modulating noise has lower and upper frequencies determined by equalities:

$$\Omega_{\text{мин}} = \Delta\Omega_{\text{ж}}, \tag{3.38}$$

$$\Omega_{\text{макс}} = 2\Omega_{\text{c}} - \Delta\Omega_{\text{ж}}. \tag{3.39}$$

This limitation is not essential, but it considerably simplifies analysis, allowing us to exclude from consideration the combination frequencies formed during the interaction of signal and interference (i.e., all components with frequencies $\Omega_c + \Omega_1$ and $\Omega_c - \Omega_1$ with fulfillment of conditions (3.38) and (3.39) will not pass through the error signal filter).

Then at the output of the selective amplifier, taking into account the introduced limitation and condition (3.36), we will have voltage

$$u_{c\,y}=\kappa_2[(1+b)m_c\cos(\Omega_c t-\varphi_c)+bm_{\text{ш}i}\cos(\Omega_{\text{ш}i}t-\varphi_i)],$$

where κ_2 - is a certain constant; $\Omega_{\text{ш}i}$ - frequency of noise component passed through the selective amplifier, where

$$|\Omega_{\text{ш}i}-\Omega_c|<\Delta\Omega_{\text{н}}.$$

At the output of the phase detector, fulfilling the function of multiplication of error signal by the reference voltage and averaging the obtained product, we have

$$u_{\phi\text{д}}=\kappa_3 U_c\{(1+b)m_c\cos\varphi_c+bm_{\text{ш}i}\cos[(\Omega_{\text{ш}i}-\Omega_c)t-\varphi_i]\}.$$

The tracking system tries to ensure equality to zero of voltage at the output of the phase detector $u_{\phi\text{д}}=0$. For this, as was indicated above, the condition of balance of amplitudes must be met

$$(1+b)m_c=bm_{\text{ш}i} \tag{3.40}$$

and of phases

$$\varphi_{\text{ш}}=\varphi_c\pm180^\circ,$$

where

$$\varphi_{\text{ш}}=(\Omega_{\text{ш}i}-\Omega_c)t-\varphi_i. \tag{3.41}$$

Expression (3.41) shows that $\varphi_{\text{ш}}$ is a slowly changing function of time, in consequence of which the point corresponding to the position of the false target shifts, which leads to shift of the equisignal line with respect to direction to target.

Quantities $\Omega_{\text{ш}i}$ and φ_i are random; therefore the law of shift of the antenna also will be random.

The value of the tracking error will be determined by the mathematical expectation of modulation factor m_c. From (3.40)

$$m_c=m_{\text{ш}i}\frac{b}{1+b}. \tag{3.42}$$

If modulation was carried out simultaneously by N harmonic oscillations with constant amplitude, then with overall 100% modulation and fixed transmitter power the modulation factor, referred to one harmonic, would be identical for all components of the modulating noise and equal to

$$m_{ni} = m_n = \frac{1}{\sqrt{N}}. \qquad (3.43)$$

For modulation by low-frequency noise this consideration is approximate. However, the essence of the process in basic features is preserved if we demand instead of equality of amplitudes equality of effective values of harmonics of modulating noise.

Substituting (3.43) in (3.42), taking into account (3.36), we obtain the final formula for m_c:

$$m_c = \sqrt{\frac{\Delta\Omega_\kappa}{\Delta\Omega_n} \frac{b}{1+b}}. \qquad (3.44)$$

Comparing (3.44) with (3.26), we find that the effectiveness of the examined barrage jamming as compared to spot jamming is $\sqrt{\frac{\Delta\Omega_n}{\Delta\Omega_\kappa}}$ times lower. Physically this is explained by the decrease of the effective modulation factor m_n of the component which gets into the passband of the coordinator $\Delta\Omega_\kappa$. To compensate for the decrease of the modulation factor by increasing interference power is impossible, since in the composition of the interference signal there is a carrier, which compensates for the action of components of the spectrum carrying false information. (This was discussed more specifically during the analysis of selecting jamming of scanning frequency).

Thus the effectiveness of barrage jamming of scanning frequency depends on the ratio noise bandwidth to the bandwidth of the coordinator. For example, if $\frac{\Delta\Omega_n}{\Delta\Omega_\kappa}$ = 100 and b → ∞, m_c = 0.1, i.e., the effectiveness of barrage jamming will be 10 times lower than the effectiveness of spot jamming.

Sweep-Through Jamming

Sweep-through jamming can be created by way of fast or slow

sweeping, for example, in sawtooth fashion (Fig. 3.14) of the frequency of the control generator, creating modulating voltage $\xi_2(t)$.

With fast sweep

$$T_{\Pi} < \tau_{K},$$

where T_{Π} – period of sweep; τ_{K} – time constant of tracking system (goniometric coordinator).

With slow sweep

$$T_{\Pi} > \tau_{K}.$$

Fast sweep. In Fig. 3.15 is shown a frequency-time diagram of ξ_2', characterizing fast sweep of control generator. With fast sweep the time of influence of the perturbing force (interference modulation) on the suppressed coordinator $\tau_{б}$ is less than the time constant of the coordinator $\tau_{K}(\tau_{б}<\tau_{K})$. However the repetition rate of disturbances is quite high, since

$$T_{б} < \tau_{K} \approx \frac{1}{\Delta\Omega_{K}},$$

where $T_{б}$ – period of fast sweep; $\Delta\Omega_{K}$ – bandpass of coordinator.

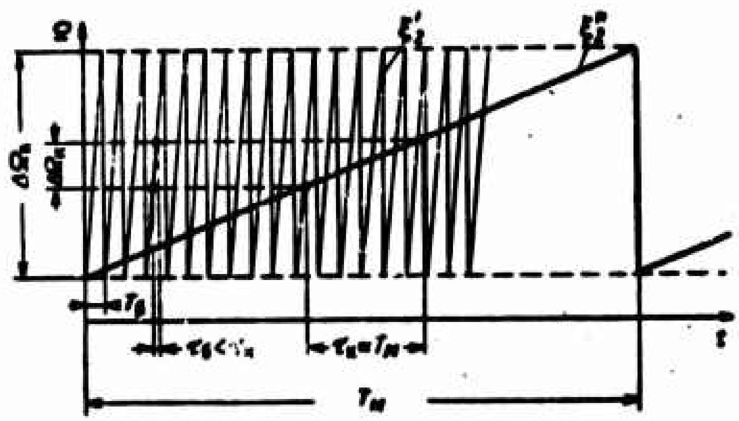

Fig. 3.15. Frequency-time diagram of modulating signal during creation of sweep-through interference with fast (ξ_2') and slow (ξ_2'') sweeps.

Let us examine the influence of sweep-through jamming with fast sweep on the goniometric coordinator. For simplicity of analysis we will consider that the sweeping of the control generator is produced stepwise (Fig. 3.16), i.e., each of the generator frequencies is

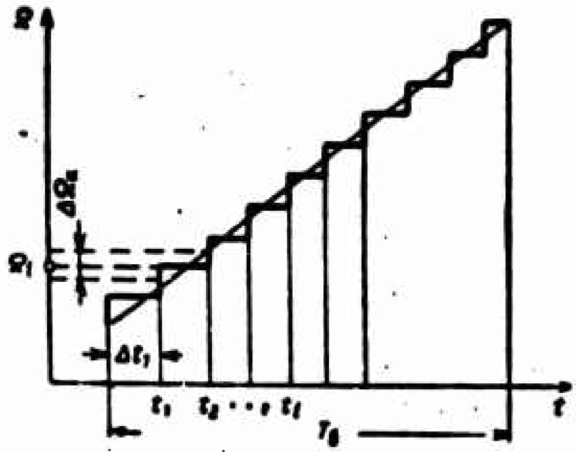

Fig. 3.16. Approximation of law of frequency shift.

formed during time

$$\Delta t = \Delta t_1 = \Delta t_2 = \ldots = \Delta t_n^* = \text{const},$$

where this formation occurs periodically with the sweep period of the generator $T_б$.

The assumption made will not change the physical principles of influence of the shown interference on the coordinator, but at the same time will simplify quantitative analysis considerably.

The modulating signal under the indicated assumption can be recorded in the following way:

$$\xi_2(t) = \sum_{i=1}^{n} [U_{ni} \cos(\Omega_i t - \varphi_{ni}) \xi(t_i)].$$

where U_{ni} – amplitude i-th modulating signal, with $U_{ni} = U_n = \text{const}$; $\xi(t_i)$ – pulse function (Fig. 3.17).

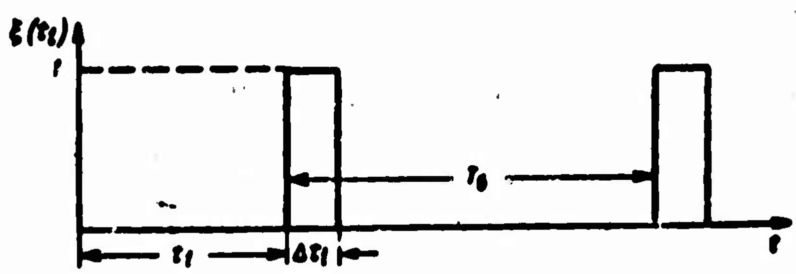

Fig. 3.17. Pulse function.

The interference signal at the point of reception can be recorded in the form

$$u_n(t) = U_n\left[1 + m_n \sum_{i=1}^{n} \cos(\Omega_i t - \varphi_{ni})\,\xi(t_i)\right] e^{j\omega t}.$$

Here m_n is interference modulation factor (made identical for all components of interference signal).

Expanding $\xi(t_1)$ in Fourier series and making necessary transpositions, taking into account filtration of all harmonics and combination frequencies different from the scanning frequency of the suppressed radar, we obtain a formula for the modulation factor of the generated interference

$$m_c = m_n \frac{\Delta\Omega_k}{\Delta\Omega_n} \frac{b}{1+b}, \qquad (3.45)$$

where $\Delta\Omega_k$ – equivalent bandpass of coordinator (or ASN system); $\Delta\Omega_n$ – sweep range of interference modulator.

The method of derivation of formula (3.45) differ in no way from that given earlier. Let us note only that during expansion of $\xi(t_1)$ in Fourier series, owing to the limitations imposed, excluding the possibility of combination frequencies getting in the passband of the error signal filter only the first member of the series should be left

$$\frac{a_0}{2} = \frac{\Delta t_i}{T_0}.$$

This permits recording the output voltage of the phase detector in the following way:

$$u_{\text{фд}} = -m_c(1+b)\cos\varphi_c + \Delta t_i/T_0\, b m_n \cos[(\Omega_c - \Omega_i)t - \varphi_{ni}].$$

Hence is obtained formula (3.45), if we equate $u_{\text{фд}}$ to zero and to use the obvious proportion

$$\frac{\Delta t_i}{T_0} = \frac{\Delta\Omega_k}{\Delta\Omega_n}.$$

Comparing formulas (3.45) and (3.44), we note that in the case of sweep-through interference with fast sweep the error in the tracking system depends to a larger degree on ratio $\Delta\Omega_{\kappa}/\Delta\Omega_{\pi}$ than in the case of creation of narrow-band blanket barrage jamming.

Formula (3.45) is valid for slower sweep speeds, when combination frequencies $\Omega_{i+1}-\Omega_i$ can be disregarded, since they do not pass through the error signal filter. With sufficiently high sweep speed combination frequencies start to play an essential role and the effectiveness of examined interference approaches the effectiveness of blanket barrage narrow-band interference.

Slow sweep. A frequency-time diagram of ξ_2'' for the case of slow sweep in shown in Fig. 3.15. For slow sweep the rate of frequency shift of modulating generator is selected on the basis of condition

$$T_{\kappa} \gg \tau_{\pi}.$$

Fulfillment of this condition ensures that time of influence of interference on the ASN system is sufficient to permit the antenna of the suppressed radar to deviate by the highest possible value, determined by formula (3.34). As a result the system will be acted upon by periodic disturbances. The approximate form of dependence of angular tracking error of covered target on time is represented in Fig. 3.18.

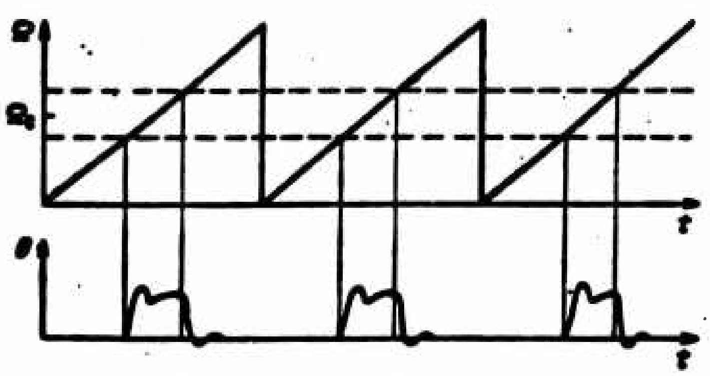

Fig. 3.18. Action of creeping interference on an ASN system.

Appraisal of the effectiveness of interferences on scanning frequency in accordance with the operational-tactical criterion can be performed by two methods:

– by way of simulation of the dynamics of guidance, interference taking into account. This method is expedient in those cases when interference signals exert direct influence on the aerodynamic control elements of rocket;

– by way of counting of misses by the formulas given in Chapter 1. This method is expedient in those cases when during the influence of interference the electronic meter (direction finder) is disconnected from the system of aerodynamic control and guidance is carried out on the basis of information coming from autonomous meters.

If during the time of action of interference the aircraft creating the interference makes a change in speed, the resultant miss a is equal to

$$a \approx \Delta_{n} - \Delta_{0},$$

where

$$\Delta_{n} = \frac{1}{2} j \sin q t_{n}^{2};$$

q – angle of attack; j – acceleration during change of speed; Δ_0 – selected miss.

3.5. Method of Formation of Jamming Signals by Means of Balanced Modulation

The basic deficiency of the selective and barrage scanning frequency jamming examined in preceding paragraphs is the compensating action of the interference carrier signal.

For this reason the maximum accessible value of modulation factor of the desired signal m_c for arbitrarily great values of b cannot exceed the modulation factor m_n of the interference signal.

Naturally, the question arises of the possibility of increasing the effectiveness of scanning frequency jamming by way of carrier

suppression. Such can be realized practically by means of balanced amplitude modulation by the scanning frequency, resulting in suppression of the carrier of the amplitude-modulated signal. For pure balanced modulation the a-m signal is recorded in the form

$$u_{\text{п}}(t) = U_{\text{п}} \cos \Omega_{\text{з}} t e^{j\omega t}.$$

The application of pure balanced modulation, as will be shown below, does not lead, in general to absolute increase of the modulation factor m_c for all values of b. The biggest values of m_c correspond to a relatively narrow range of values of b, close to unity.

Best results are obtained with combined modulation, combining carrier suppression with amplitude modulation at the scanning frequency of sideband components of the spectrum of the interference signal.

Let us determine the value of modulation factor m_c during the influence on the tracking system of desired signal

$$u_c(t) = \dot{U}_c e^{j(\omega t - \psi_c)}$$

and interference signal formed by way of combined modulation

$$u_{\text{п}}(t) = U_{\text{п}} [1 + m_{\text{п}} \cos(\Omega_{\text{п}} t - \varphi_{\text{п}1})] \cos(\Omega_{\text{з}} t - \varphi_{\text{з}2}) e^{j(\omega t - \psi_{\text{п}})}.$$

Subsequently we will assume $\psi_c = \psi_{\text{п}} = 0$, which in first approximation is permissible in view of the comparative brief time of coherence of useful and interference signals. Furthermore, it is considered that $\varphi_{\text{з}1} = \varphi_{\text{з}2} = \varphi_{\text{з}}$.

After passing through the scanning antenna the signal at the receiver input can be recorded in the following way:

$$u_{\text{вх}} = \varkappa_1 U_c \{1 + b[1 + m_{\text{п}} \cos(\Omega_{\text{п}} t - \varphi_{\text{п}})] \cos(\Omega_{\text{з}} t - \varphi_{\text{з}})\} \times$$
$$\times [1 + m_c \cos(\Omega_c t - \varphi_c)] e^{j\omega t},$$

where $\varkappa_1$ is a constant.

For the purpose of certain simplification of calculations we will consider the second detector to be of square-law type, which in this case is not essential from the stant point of final results.

Taking into account what has been said, we record the signal at the input of the selective amplifier (error signal filter) as:

$$u_{c\,\Sigma} = \kappa_2 U_c'^2 \{1 + b[1 + m_\text{н} \cos(\Omega_L t - \varphi_\text{н})] \cos(\Omega_\text{н} t - \varphi_\text{н})\}^2 \times$$
$$\times [1 - m_c \cos(\Omega_c t - \varphi_\text{н})]^2.$$

where κ_2 is a constant.

To the input of the phase detector, following the selective amplifier will pass members containing the first harmonic of the scanning frequency Ω_c . Therefore (when $\Omega_\text{н} = \Omega_c$)

$$u_{\phi\,\text{д}\,\text{вх}} = \kappa_3 U_c'^2 \left[-m_c \left(2 + \frac{3}{2} b^2\right) - m_c m_\text{н}^2 \frac{3b^2}{4} + \right.$$
$$\left. + m_c^2 \frac{3b}{2} - 3 m_c m_\text{н} b + m_\text{н} \frac{3b^2}{2} + \frac{5}{4} m_c^2 m_\text{н} b^2 - \frac{m_\text{н}^2 m_c^2}{2} b^2 + 2b \right], \qquad (3.46)$$

where κ_3 is a constant.

The voltage at the output of the phase detector is obtained after multiplication of input signal $u_{\phi\,\text{д}\,\text{вх}}$ by reference voltage $u_{\text{оп}} = \cos \Omega_c t$ and filtration of all harmonics but the constant component. This voltage in steady state is held close to zero. Making use of this circumstance, are obtain and equation for determination of m_c:

$$A m_c^2 - B m_c + C = 0, \qquad (3.47)$$

where

$$A = \frac{3}{2} b + \frac{5}{4} m_\text{н} b^2;$$

$$B = 2 + \frac{3}{2} b^2 + 3 m_\text{н} b + \frac{5}{4} m_\text{н}^2 b^2;$$

$$C = 2b + \frac{3}{2} b^2 m_\text{н}.$$

Results of solution of equation (3.47) are shown in Fig. 3.19 in the form of a family of curves $m_c = m_c(b)$, where $m_\text{н}$ is the parameter of the family. Curve 1 corresponds to case $m_\text{н} = 0$, i.e., pure balanced

modulation.

The family of functions $m_c(b)$ for the case of combined modulation is represented in Fig. 3.19 by curves 2, 3, 4, and 5, plotted respectively for values of modulation factor m_n = 0.2, 0.4, 0.8, and 1. The broken lines on these curves show the ranges of values of b within the limits of which, owing to the adopted assumptions, equaltion (3.47) has complex roots. These circumstances are caused by overshoot of curves beyond the level m_c = 1.

For comparison in Fig. 3.19 there also is plotted a family of functions $m_c = m_c(b)$ (curves 6, 7, 8, and 9), calculated by formula (3.26) for earlier shown values of modulation factor m_n (0.2, 0.4, 0.8, and 1).

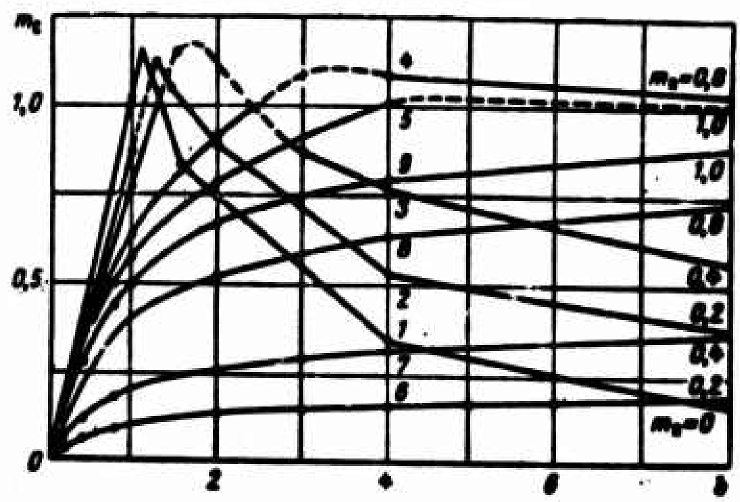

Fig. 3.19. Dependence of modulation factor of signal at ASN receiver input on signal-to-noise ratio during the action of spot jamming with balanced modulation.

In order to create the barrage jamming signal, it is necessary either sequentially or simultaneously to carry out balanced modulation by means of a certain conbination of frequencies of assigned range.

In the case of sequential realization of pure balanced modulation the signal at the output of the scanning antenna of the suppressed radar will be recorded in the following way:

$$u = \varkappa_1 U_c \left[1 + b\sum_{i=1}^{N} \cos(\Omega_i t + \varphi_{ni})\,\xi(t_i)\right][1 + m_c \cos(\Omega_c t - \varphi_c)]\, e^{j\omega t},$$

where $\xi(t_i)$ is pulse function:

$$\xi(t_i) = \begin{cases} 1 \text{ for } t_i < t < t_i + \Delta t_i, \\ 0 \text{ for } 0 < t < t_i \text{ и } t_i + \Delta t_i < t < t_i + T - \Delta t_i, \end{cases}$$

$$t_i = \sum_{k=1}^{i} \Delta t_k.$$

Pulse function $\xi(t_i)$ can be expanded in Fourier series in harmonics of frequency $\Omega = 2\pi/T$:

$$\xi(t_i) = \frac{\Delta t_i}{T} + \sum_{k=1}^{\infty} \frac{1}{\pi}\left\{\left(\frac{1}{k}\sin\frac{2\pi k\Delta t_i}{T}\right)\cos k\Omega t + \right.$$
$$\left. + \left[\frac{1}{k}\left(1 - \cos\frac{2\pi k\Delta t_i}{T}\right)\right]\sin k\Omega t\right\}.$$

We will make a series of assumptions with respect to the spectrum of modulating oscillations. Let us, as earlier, consider

$$\Delta\Omega_n \ll \Omega \ll \Omega_i.$$
$$|\Omega_{i+1} - \Omega_i| = \Delta\Omega_n,$$

where $\Delta\Omega_n$ is width of spectrum covered by sequence of modulating frequencies Ω_i $(i = 1, 2, 3, \ldots)$:

$$\Delta\Omega_n = |\Omega_n - \Omega_1|.$$

Thus it is assumed that the passband of the selective amplifier simultaneously includes one signal of frequency Ω_1.

The assumptions made do not change the fundamental essence of the question, but in significant measure simplify the process of calculations, owing to the possibility of exclusion from consideration of a large number of combination frequencies $(\Omega_i \pm \Omega_j) \pm (k+m)\Omega$ $(i = 1, 2, 3, \ldots, N;$ $j = 1, 2, 3, \ldots, N;$ $m = 1, 2, 3, \ldots;$ $k = 1, 2, 3, \ldots)$.

Then at the output of the selective amplifier, taking into account the above assumptions, we have

$$u_{\Phi\,\text{вых}} = \varkappa_s U_c^2 \{2m_c A \cos(\Omega_c t - \varphi_c) + \\ + 2b\frac{\Delta t_i}{T}\left(1+\frac{m_c^2}{2}\right)\cos(\Omega_i t - \varphi_{ui}) + \\ + \frac{1}{2}bm_c^2\frac{\Delta t_i}{T}\cos[(2\Omega_c - \Omega_i)t - (2\varphi_c - \varphi_{ui})] + \\ + \frac{1}{2}bm_c^2\left(\frac{\Delta t_i}{T}\right)^2\cos[(2\Omega_i - \Omega_c)t - (2\varphi_{ui} - \varphi_c)]\}, \tag{3.48}$$

where

$$A = 1 + \frac{b^2}{2}\left[\left(\frac{\Delta t_i}{T}\right)^2 + \frac{1}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{k^2}\left(1 - \cos\frac{2\pi k\Delta t_i}{T}\right)\right] = \\ = 1 + \frac{1}{2}b^2\frac{\Delta t_i}{T}.$$

With $\Omega_i = \Omega_c$, $\varphi_c = \varphi_u \pm \pi$, $\Delta t_i = T$ voltage $u_{\Phi\,\text{вых}}$, determined by formula (3.48), naturally coincides with voltage $u_{\Phi\,\text{вых}}$, determined by formula (3.46), if in the latter we assume $m_s = 0$ (balanced modulation).

Equating the voltage at the output of the phase detector to zero, under condition $\Omega_i = \Omega_c$ and $\varphi_c = \varphi_u \pm \pi$, we obtain the following fundamental equation for determination of m_c in steady-state operating conditions with successive emission of barrage jamming signals:

$$\frac{3}{2}b\frac{\Delta t_i}{T}m_c^2 - 2\left[1 + \frac{1}{2}b^2\left(\frac{\Delta t_i}{T} + \frac{1}{2}\frac{\Delta t_i^2}{T^2}\right)\right]m_c + 2b\frac{\Delta t_i}{T} = 0.$$

Hence is found the optimum value of b, for which m_c is maximum

$$b_{\text{опт}} = \frac{2N}{\sqrt{2N+1}},$$

and accordingly,

$$m_{c\,\text{макс}} = \frac{2}{3}(\sqrt{2N+1} - \sqrt{2N-2}). \tag{3.49}$$

Here

$$N = \frac{T_s}{\Delta t_i} \approx \frac{\Delta\Omega_s}{\Delta\Omega_s}.$$

In general successive emission of interference signals

$$m_c^{(n)} = \frac{2}{3}\left[\left(\frac{N}{b}+\frac{b}{4}\frac{2N+1}{N}\right)-\sqrt{\left(\frac{N}{b}+\frac{b}{4}\frac{2N+1}{N}\right)^2-3}\right]. \tag{3.50}$$

If the interference signal is formed by way of simultaneous radiation of all modulating frequencies from range Ω_1 to Ω_N, the input of the selective amplifier, after obvious conversions in the receiver, will be fed voltage (with pure balanced modulation)

$$u_{c\,y} = k_y U_c^2\left[1+b\sum_{l=1}^{q}\cos(\Omega_l t+\varphi_l)\right]^2 [1+m_c\cos(\Omega_c t-\varphi_c)]^2.$$

Considering the assumptions made earlier about the structure of the spectrum of the modulating function $(\Omega_{l+1}-\Omega_l=\Delta\Omega_s,\ \Delta\Omega_s\ll\Omega\ll\Omega_s)$ valid, we arrive finally at the following to equation for $m_c^{(0)}$ for simultaneous radiation of barrage jamming signals[1]

$$\frac{3}{2}bm_c^2-2\left[1+\frac{1}{4}b^2(2N+1)m_c+2b\right]=0.$$

Accordingly

$$b_{0,y}=\frac{2}{\sqrt{2N+1}}.$$

The formula for $m_c^{(0)}$ is obtained just as in the case of successive emission of interference signals (3.49).

In the general case of simultaneous radiation of interference signals

$$m_c^{(0)}=\frac{2}{3}\left\{\left[\frac{1}{b}+\frac{1}{4}b(2N+1)\right]-\sqrt{\left[\frac{1}{b}+\frac{1}{4}(2N+1)\right]^2-3}\right\}. \tag{3.51}$$

Results of calculations by formulas (3.50) and (3.51) are shown in Fig. 3.20 in the form of a family of curves for $m_c(b)$. The parameter of the family is

$$N=\frac{T}{\Delta t_s}\approx\frac{\Delta\Omega_s}{\Delta\Omega_c}.$$

---

[1]It is considered that $\Omega_1=\Omega_c$, i.e., a stable equilibrium of system is possible, with which $\varphi_c=\varphi_s\pm180°$.

(N = 10 on curves 1 and 2; N = 4 on curves 4 and 5). Here is given a family of curves of $m_c(b)$ (curves 3 and 6) for amplitude modulation of radiated signal by narrow-band noise, calculated by formula (3.45).

Fig. 3.20. Dependence of modulation factor of signal at input of ASN receiver on signal-to-noise ratio during the action of barrage jamming with balanced modulation.

Accordingly, parameter $(m_s = 1/\sqrt{N}$ was selected equal to 1/2 curve 6) and $1/\sqrt{10}$ (curve 3).

Curves 1, 4 and 2, 5 determine $m_c(b)$ respectively for simultaneous and successive emission of balanced-modulated interference signals.

Of interest is comparative appraisal of the limiting possibilities of increasing m_c by means of barrage jamming signals with balanced and amplitude modulations by narrow-band noise for any values of N.

Let us introduce designation

$$\eta = \frac{m_c^{(б)}}{m_c^{(a)}},$$

where $m_c^{(б)}$ and $m_c^{(a)}$ are determined by formulas (3.49) and (3.44); in the last formula it is necessary to set $b = \infty$.

Then with the help of (3.49) and (3.44) we find

$$\eta = \frac{2}{3}\sqrt{N}(\sqrt{2N+1} - \sqrt{2N-2}).$$

If N >> 1, then

$$\eta = \frac{1}{\sqrt{2}}.$$

Thus amplitude modulation by narrow-band noise provides approximately $\sqrt{2}$ times larger values of errors in angular tracking than balanced modulation in the barrage variant for the same value of N, but maximum value of error in the case of balanced modulation is provided by smaller values of b.

3.6. Intermittent Jamming

Intermittent jamming signals consists of a periodic sequence of high-power radio pulses of defined duration and frequency, radiated by a jamming transmitter aboard the covered aircraft (Fig. 3.21). The action of intermittent interference on radar with scanning antenna is based on the use of the transient conditions existing in a amplifier with adjustable gain during influence on it of pulse signals.

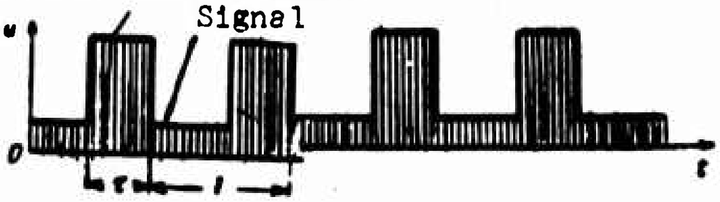

Fig. 3.21. Intermittent jamming.

In Fig. 3.22 is presented the diagram of the i-f amplifier of the receiver of a radar with scanning antenna. The gain of the i-f amplifier is regulated by an AGC system. If the amplitude of the signal at the output of the i-f amplifier does not exceed the value of delay voltage E_3, the AGC system is open-circuited and the gain of the receiver does not change. The AGC system regulates gain when the amplitude of the output voltage exceeds that of the delay voltage.

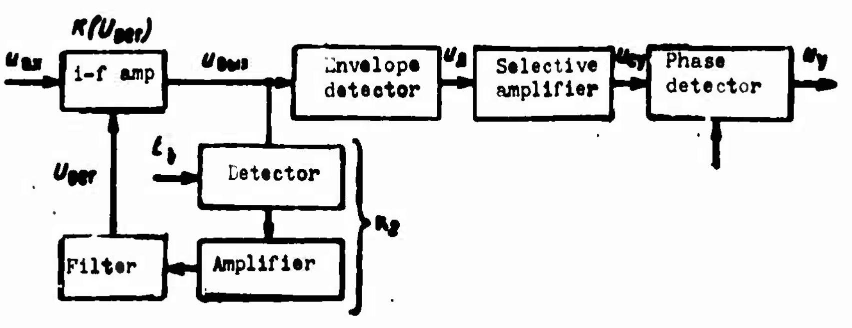

Fig. 3.22. Block diagram of receiver with AGC system.

In normal state the AGC system tries to keep the output amplitude constant. However, its time constant is so selected as to exclude the possibility of amplitude demodulation of the signal, which inevitably would lead to considerable loss of information in the process of signal conversion in the receiver. The undistorted amplitude-modulated signal, whose modulation percentage is proportional to angular error, is depicted in Fig. 3.23a.

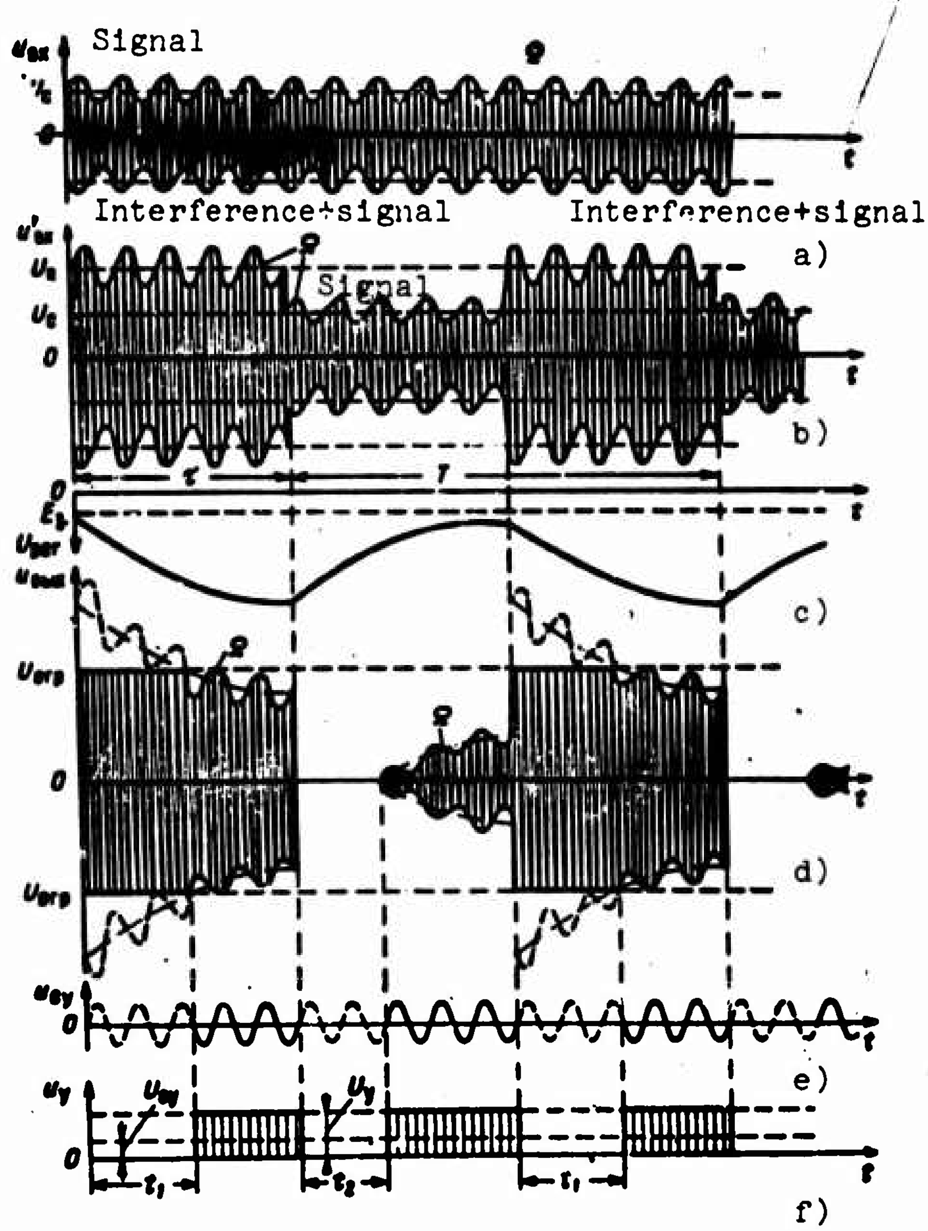

Fig. 3.23. Action of intermittent interference on ASN system with sequential comparison of signals: a) signal at receiver input in the absence of interference; b) signal and interference at receiver input; c) AGC control voltage; d) voltage at receiver output; e) signal at output of selective amplifier; f) control voltage (at output of phase detector).

During the influence of intermittent interference the voltage at the i-f input will have the form depicted in Fig. 3.23b. The envelope of the input signal U_0 is changed abruptly, but the gain is unable to change abruptly: it is changed in accordance with change of control voltage U_{per}, on the grids of the i-f tubes (Fig. 3.23c). Since the gain of the i-f amplifier at the time of arrival of a high-power noise pulse (t= 0) was determined by the comparatively small amplitude of the desired signal and was maximum, the noise at certain time t_1 exceeds the level of limitation $U_{огр}$ and amplitude modulation of the interference signal, caused by scanning of the receiving antenna, will be cutt off (Fig. 3.23d). The control signal at the output of the selective amplifier $u_{су}$ and phase detector $u_{фд}$ is absent, and the ASN system will be open-circuited for time t_1 (Fig. 3.23e, f). The value of time t_1 is determined by the duration of transient conditions in the AGC system, i.e., the time during which control voltage U_{per}, and consequently the i-f gain, decrease to such values that the noise exceeds the level of limitation and the ASN system is reactivated.

After termination of the noise pulse the gain of the receiver will be minimum, so that during a certain interval of time t_2 the weak desired signal will not pass through the receiver. The ASN system will again be open-circuited for time t_2. This time is determined by the duration of rise of i-f gain (increase of U_{per}) to a level ensuring reliable reception of desired signal (Fig. 3.23d).

Thus in one cycle during the influence of intermittent interference the ASN system will be open-circuited twice:

– the first time for time t_1, as a result of cut off of useful amplitude modulation of interference signal in the overloaded receiver;

– the second time for time t_2, owing to the absence of reception of the weak signal by the receiver with low gain.

The values of time intervals t_1 and t_2, during which the ASN system is inactive, depend both on the parameters of the AGC system and on characteristics of the interference. In first approximation they can be determined by the following formulas [35]:

$$t_1 = -T_\phi \ln\left[1 - \frac{K_\phi (U_n - U_{огр})}{a\kappa_3 U_n (U_{огр} - E_3)}\right],$$
$$t_2 = T_\phi \ln \frac{U_c a\kappa_3 (K_\phi U_n - E_3)}{(1 + U_c a\kappa_3)(K_\phi U_c - E_3)},$$

where T_ϕ – time constant of AGC feedback filter; K_ϕ, a, κ_3 – parameters of AGC system (see 3.2); U_n – mean value of noise envelope at i-f input; $U_{огр}$ – level of limitation of receiver; E_3 – AGC delay voltage.

Under ordinary conditions, without intermittent interference, to a given value of angular error θ corresponds a defined value of control signal U_y (Fig. 3.23f). The influence of interference leads to decrease of control voltage to a level determined by the mean value of error pulse voltage U_{0y} (value of angular displacement θ in both cases is assumed identical). This is equivalent to decrease of the transfer function of the ASN system, which in first approximation can be determined by the formula

$$\bar{k}_{ny} = \frac{k_{ny}}{Q}, \tag{3.52}$$

where k_{ny} – transfer function of the ASN system in the absence of intermittent interference; Q – off-duty factor of entry of information.

From Fig. 3.23f

$$Q = \frac{T}{T - (t_1 + t_2)},$$

therefore expression (3.52) can be rewritten in the form

$$\bar{k}_{ny} = \left(1 - \frac{t_1 + t_2}{T}\right) k_{ny}.$$

In Fig. 3.24 is depicted qualitative dependence of transfer function of the ASN system on the ratio of noise pulse duration to the time constant of the filter τ/T_ϕ for constant values of pulse repetition period T and amplitude of interference U_n. For certain optimum values of $\tau_{опт}$ and $T_{опт}$ the transfer function of the ASN system becomes minimum.

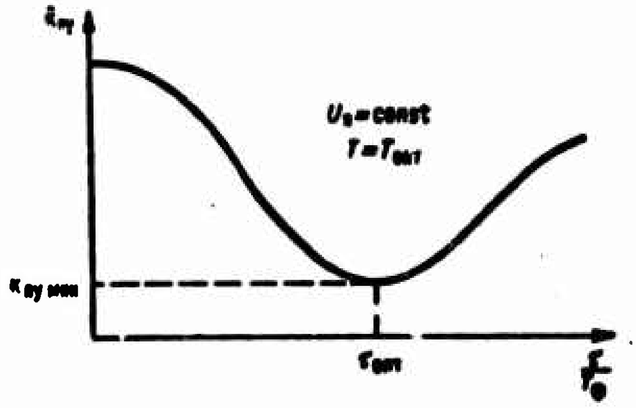

Fig. 3.24. Influence of parameters of intermittent interference on transfer function of direction finding device of the ASN system.

Decrease or increase of pulse duration τ and period T with respect to values of $\tau_{опт}$ and $T_{опт}$ leads to decrease of effectiveness of interference.

In first case this is due to the fact that the AGC system is unable to respond to every noise pulse and set the gain of the receiver in accordance with the mean value of intensity of interference and signal, so that cutoff of amplitude modulation and suppression of desired signals will be absent.

In the second case the AGC system, conversely, is able to work in accordance with useful and interference signals. Intervals of time of inactive state of the ASN system will decrease and periods of active state increased, which leads in the end to lowering of effectiveness of the examined influence of interference.

Decrease of the transfer function of the ASN system under the influence of intermittent interference leads to growth of dynamic target tracking error. For a linearly increasing input signal form $\theta_{вх}=vt$, where v is rate of change of input angle, the dynamic target tracking of error system with astaticism of first order is changed in inverse proportion to $K_{ву}$, i.e.,

$$\delta_{д}=\frac{v}{K_{ву}}.$$

Intermittent interference may even cause loss of stability in a certain class of ASN systems with so-called "leak-shaped" amplitude-phase characteristic. For such systems loss of stability can occur both with increase and decrease of transfer function κ_{ay}.

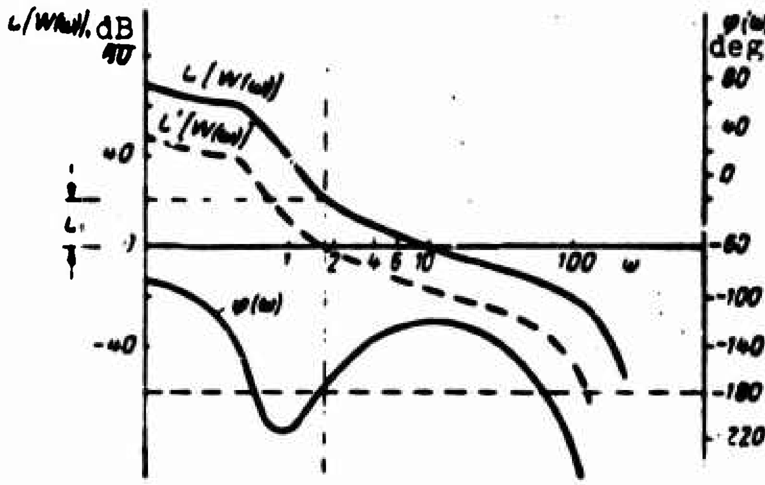

Fig. 3.25. Logarithmic characteristics of ASN system.

In Fig. 3.25 are given amplitude $L[W(\omega)]$ and phase $\varphi(\omega)$ logarithmic characteristics of a hypothetical ASN system [36]. The examined system will become unstable if during the influence of intermittent interference the transfer function κ_{ay} decreases by $L_1 \approx -20$ dB. Knowing the critical value of transfer function κ_{ay}, at which loss of stability occurs, it is possible from Fig. 3.24 to determine the stability of system in accordance with parameter κ_{ay} on plane with parameters "duration τ-repetition period T" of noise pulses.

CHAPTER 4

ACTIVE JAMMING OF AUTOMATIC TRACKING SYSTEMS WITH SIMULTANEOUS COMPARISON OF SIGNALS

4.1. Introduction

The ASN system with simultaneous comparison of signals (monopulse systems) have recently found wide use. They have higher noise immunity with respect to active jamming created from one point.

Owing to the principle of monopulse direction finding interference from one point can effectively exert influence only on imperfect systems, for example ASN systems with nonidentical amplitude or phase characteristics [35, 38, 41, 42]. Since the influence of interference from one point on monopulse radar has been studied in sufficient detail, and this type of jamming is little effective, we will not stop on its analysis.

Below is examined interference, created by a remote source (interference from two and more points in space). In the simplest case interference of such kind can constitute a pair spatially separated reflecting objects. In the presence in the field of vision of a radar of several sources of interference "multipoint" interference is realized.

Determining value during the analysis of influence of "two-point" jamming is taken on by phase relationships of signals reaching the input of the radar. Therefore we will subsequently separate interference signals into incoherent and coherent types. The first are characterized

by absence of any definite relationship between phases of high-frequency oscillations of both interference signals, reaching the input of the system, whereas the difference of phases of high-frequency oscillations corresponding to coherent interference signals remains constant for a relatively long time.

4.2. Direction Finding Characteristics of ASN Systems with Simultaneous Comparison of Signals

At present there are quite a few different ASN systems with simultaneous comparison of signals. Most widely used is the system with sum-difference treatment of signal. We will explain the principle of construction of such systems briefly and examine their direction finding characteristics.

In Fig. 4.1 is presented a functional diagram of the direction finder of an ASN system of mixed (amplitude-phase) type. From the given circuit phase and amplitude direction finders can be obtained as individual cases. For example, when $\theta_0 = 0$ and $\psi_0 = \pi/2$ we have a pure phase ASN system.

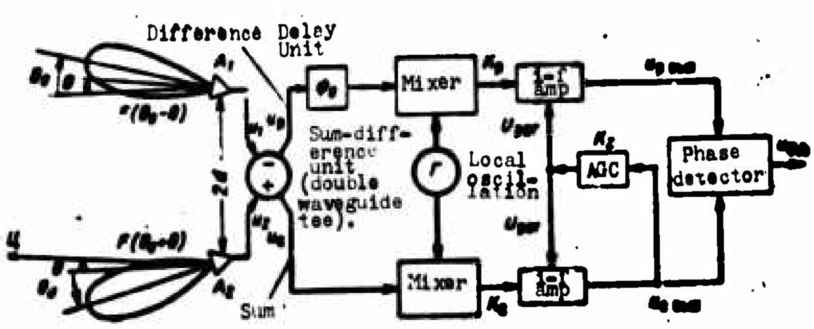

Fig. 4.1. Functional diagram of a monopulse type with sum-difference treatment of signal (amplitude-phase type).

Signals received from target Ц by antennas A_1 and A_2 are fed to the circuit forming the sum (u_c) and difference (u_p) of voltages. With angular displacement of target from the equisignal direction by angle θ these voltages are equal respectively to

$$u_c = u_1 + u_2 = U[F(\theta_0 - \theta)\cos(\omega t + \varphi) + F(\theta_0 + \theta)\cos(\omega t - \varphi)], \tag{4.1}$$

$$u_p = u_1 - u_2 = U[F(\theta_0 - \theta)\cos(\omega t + \varphi) - F(\theta_0 + \theta)\cos(\omega t - \varphi)]. \tag{4.2}$$

Here U – amplitude of signal at input of antenna system (A_1 and A_2);

$F(\theta)$ – standardized radiation pattern of antenna (A_1 or A_2);

θ_0 – angular displacement of peaks of antenna radiation pattern relative to the equisignal direction;

ω – signal frequency;

$\phi = kd \sin \theta$ – phase shift in antennas A_1 and A_2 caused by displacement of target by angle θ relative to the equisignal direction;

$k = 2\pi/\lambda$ – wave coefficient;

2d – distance between phase centers of antennas A_1 and A_2.

At the i-f of the sum and difference channels we obtain

$$u_{c\,вых} = K_c U[F(\theta_0 - \theta)\cos(\omega_{пр} t + \varphi) + F(\theta_0 + \theta)\cos(\omega_{пр} t - \varphi)], \tag{4.3}$$

$$u_{p\,вых} = K_p U[F(\theta_0 - \theta)\cos(\omega_{пр} t + \varphi - \psi'_0) - F(\theta_0 + \theta)\cos(\omega_{пр} t - \varphi - \psi'_0)], \tag{4.4}$$

where K_c and K_p – gains of sum and difference channels respectively;

$\omega_{пр}$ – intermediate frequency;

$\psi'_0 = \psi_0 + \phi_c - \phi_p$ – phase shift, consisting of phase shift in phase inverter of difference channel (ψ_0) and in i-f's of sum (ϕ_c) and difference (ϕ_p) channels.

Subsequently it is considered that sum and difference channels are completely identical ($\phi_c = \phi_p$); therefore

$$K_c = K_p = K, \quad \psi' = \psi_0. \tag{4.5}$$

In steady-state operating conditions the gain of the i-f of the

sum channel, in accordance with formula (3.8), can be recorded in the form

$$K_{\text{c}} = \frac{K_0}{1 + \alpha \varkappa_2 U_{\text{c вх}}}, \tag{4.6}$$

where

$$U_{\text{c вх}} = U\left[F^2(\theta_0 - \theta) + F^2(\theta_0 + \theta) + 2F(\theta_0 - \theta)F(\theta_0 + \theta)\cos 2\varphi\right]^{\frac{1}{2}} \tag{4.7}$$

or taking into account (4.5)

$$K = K_{\text{c}} = K_{\text{p}} = \frac{K_0}{1 + \mu\left[F^2(\theta_0 - \theta) + F^2(\theta_0 + \theta) + 2F(\theta_0 - \theta)F(\theta_0 + \theta)\cos 2\varphi\right]^{\frac{1}{2}}}, \tag{4.8}$$

where $\mu = \alpha\varkappa_2 U$ - is equivalent transfer function of gain of the AGC system.

For amplitude type direction finding in formulas (4.3) and (4.4) one should set $\phi = 0$ or $d = 0$ (radiation patterns of antennas A_1 and A_2 are combined) and $\psi' = 0$; with phase type direction finding $\theta_0 = 0$ (radiation patterns of antennas A_1 and A_2 are spaced, with directions of their peaks are parallel) and $\psi_0 = \pi/2$.

Assuming that the phase detector executes the operation of multiplication and averaging of input signals, we find, taking into account relationships (4.3) and (4.4), the voltage at the output of the direction finder[1]

$$u_{\phi \text{д}} = \varkappa' \overline{u_{\text{c}} u_{\text{p}}} = \frac{\varkappa'^2}{2} K_{\text{c}} K_1 U^2 \times \times \{[F^2(\theta_0 - \theta) - F^2(\theta_0 + \theta)]\cos\psi_0 + 2F(\theta_0 - \theta)F(\theta_0 + \theta)\sin 2\varphi \sin\psi_0\}. \tag{4.9}$$

[1]Let us remember that the averaging operation in the given case physically means filtration by the filter of the phase detector of high-frequency components of signal $u_{\phi\text{д}}$.

where k' – is a constant.

Relationships (4.8) and (4.9) give expressions for direction finding characteristics.

Amplitude Direction Finding

Considering in (4.9) $\phi = 0 (d = 0)$, $\psi_0 = 0$ and substituting instead of K its expression (4.8), we obtain a formula for the direction finding characteristic of the amplitude ASN system

$$(u_{\text{фд}})_A = k_{\text{фд}} \frac{\mu^2 [F^2(\theta_0-\theta) - F^2(\theta_0+\theta)]}{\{1+\mu [F(\theta_0-\theta) + F(\theta_0+\theta)]\}^2}, \tag{4.10}$$

where

$$k_{\text{фд}} = \frac{K_0^2 \kappa'}{2a^2 \kappa_2^2}.$$

Phase Direction Finding

In the case of the phase ASN system in relationship (4.9) it is necessary to set $\theta_0 = 0$ and $\psi_0 = \pi/2$. Then, taking into account equality (4.8), we obtain

$$(u_{\text{фд}})_{\text{ф}} = k_{\text{фд}} \frac{2\mu^2 F^2(\theta) \sin 2\varphi}{[1 + 2\mu F(\theta) \cos\varphi]^2}. \tag{4.11}$$

For convenience of comparison of direction finding characteristics with phase and amplitude direction finding we will consider that maximum dimensions of antenna system in both cases is identical and equal to D. Furthermore, we will assume that the phase centers of antennas (i.e., their radiators) are spaced by an amount 2d = D/2, and the diameter of each reflector (for example, parabolic) is equal to D/2 (Fig. 4.2).

Under these conditions formula (4.11) is converted to form

$$(u_{\text{фд}})_{\text{ф}} = k_{\text{фд}} \frac{2\mu^2 F^2(\theta) \sin x}{\left[1 + 2\mu F(\theta) \cos\frac{x}{2}\right]^2}. \tag{4.12}$$

where

$$x=\frac{\pi D_1}{\lambda}\sin\theta=\frac{4\pi d}{\lambda}\sin\theta.$$

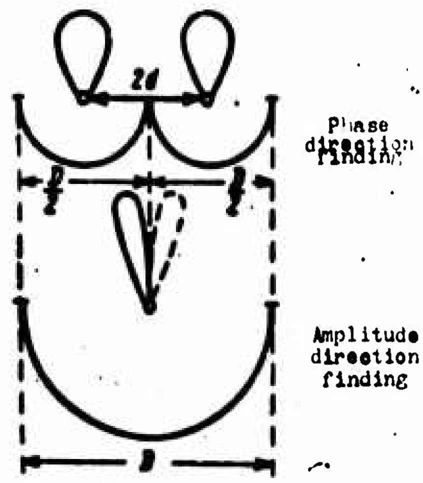

Fig. 4.2 Antenna arrangements of monopulse ASN systems of phase and amplitude types.

In Figs. 4.3 and 4.4 are presented direction finding characteristics of amplitude and phase ASN systems in the form of family $u_{\phi x}=u_{\phi x}(\theta,\mu)$[1].

Characteristics were plotted in accordance with formulas (4.10) and (4.12) for different values of parameter μ (μ = 1, 3, 10), proportional to intensity of input signal U. The antenna radiation pattern was approximated during the plotting by function

$$F(\theta)=\frac{\sin\frac{\pi D'}{\lambda}\theta}{\frac{\pi D'}{\lambda}\theta}=\frac{\sin x}{x}, \qquad (4.13)$$

---

[1]For simplicity we take $k_{\phi x}=1$; $\theta_{0.5}\approx\frac{\lambda}{D}$ — is half-power width of antenna radiation pattern.

where D' = D in the case of amplitude direction finding;

D' = 2d in the case of phase direction finding (Fig. 4.2).

The direction finding characteristic of the amplitude ASN system was plotted for a value of angular displacement of peaks of antenna radiation pattern of $\theta_0 = \theta_{0.5}/3$. For other values of θ_0 characteristics have analogous form.

Analysis of graphs (Figs. 4.3 and 4.4) and expressions (4.10) and (4.12) shows that direction finding characteristics in considerable degree depend on amplitude of received signal U ($\mu = ak_2U$).

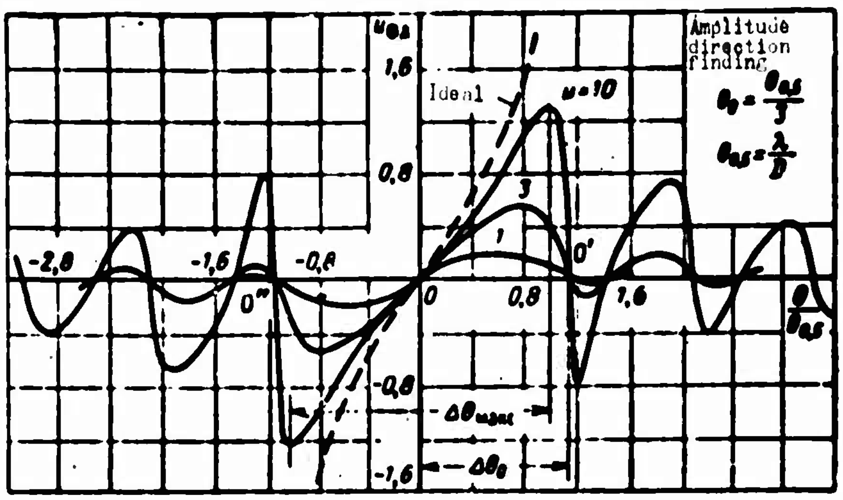

Fig. 4.3. Direction finding characteristics of amplitude ASN systems with simultaneous comparison of signals and sum-difference treatment.

For large amplitudes of input signals ($\mu \to \infty$) the direction finding characteristics become idealized (dotted lines), and formulas for them are found in work [42]:

$$(u_{\phi\,\text{д}})_A = \kappa_{\phi\,\text{д}} \frac{F(\theta_0 - \theta) - F(\theta_0 + \theta)}{F(\theta_0 - \theta) + F(\theta_0 + \theta)},$$

$$(u_{\phi\,\text{д}})_\phi = \kappa_{\phi\,\text{д}} \operatorname{tg}\left(\frac{\pi D}{\lambda} \sin\theta\right).$$

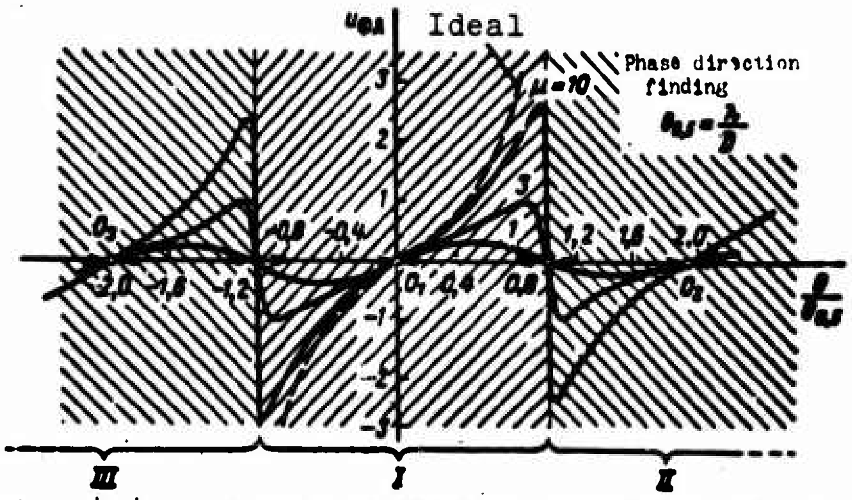

Fig. 4.4. Direction finding characteristics of phase type ASN systems with simultaneous comparison of signals and sum-difference treatment.

Direction finding characteristics are linear only for small angles θ. All direction finding characteristics have false equisignal directions (slide lobes). In connection with this, depending upon the value of angle θ, it is possible to split the direction finding characteristics into several zones of stable target tracking.[1] Zone I (Fig. 4.4) we will call the main zone (and its lobes, major lobes) of the direction finding characteristic, while zones II and III are false zones (minor side lobes).

If in any of the zones of stable tracking a sufficiently powerful source of electromagnetic waves appears, the ASN system tracks it in accordance with equisignal direction, corresponding either to the major lobe of the direction finding characteristic (zone 1, point 0_1) or its minor lobes (points 0_2, 0_3 zones II. III). In the last case the guidance system (homing guidance) will be fed considerable errors on target position data.

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[1]Zones of stable tracking are determined by the presence in them points of stable equilibrium. In this case these points are 0_1, 0_2, 0_3, ... (transfer function of system at these points is positive).

As can be seen from the figure, the width of the major lobe of the direction finding characteristic with respect to nulls $\Delta\theta_0$ does not depend on signal power and is determined only by the radiation pattern of the antenna of the direction finder and the value of angle θ_0. Signal power renders considerable influence on distance between peaks of the direction finding characteristic and its slope. A graphic representation of this is given in Fig. 4.5, where along the axis of ordinates is plotted the ratio of distance between peaks $\Delta\theta_{макс}$ to the half-power width of the radiation pattern $\theta_{0.5}$, taken during calculations equal to λ/D. Solid lines correspond to amplitude direction finding, dotted to phase.

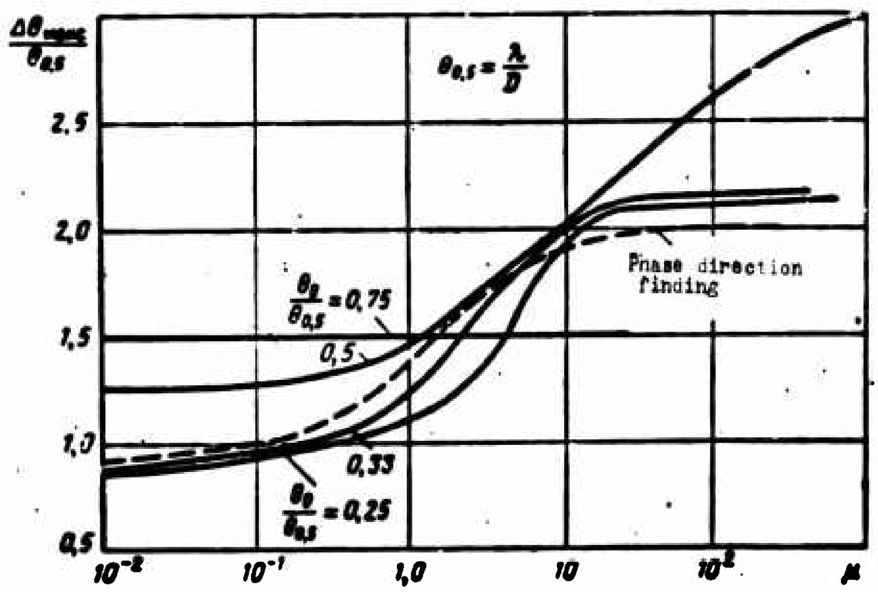

Fig. 4.5. Dependence of angular distance between peaks of main zone of direction finding characteristic on signal intensity.

Dependence of slope of direction finding characteristic on signal intensity is shown in Fig. 4.6.

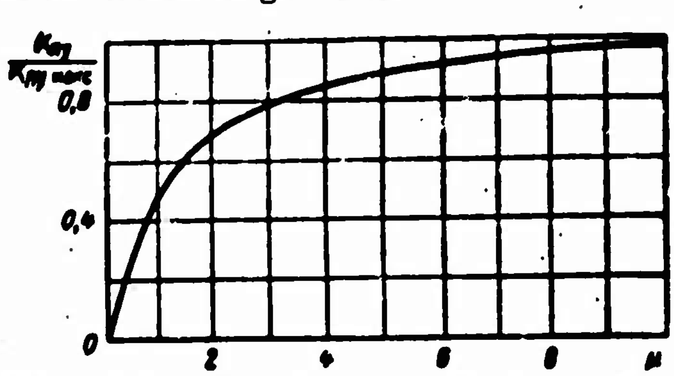

Fig. 4.6. Dependence of slope of direction finding characteristic on signal intensity.

Let us note that the direction finding characteristics of amplitude ASN systems with simultaneous and sequential comparison of signals are described by different formulas, (3.10) and (4.10). This is connected with the fact that formation of the direction finding characteristic of amplitude ASN systems with simultaneous comparison is influenced by both amplitude and phase characteristics of antenna, whereas for ASN systems with sequential comparison the phase characteristic of the antenna does not influence the direction finding characteristic. In the last case phase distinctions in signals picked up by oppositely phased lobes of the radiation pattern of the scanning antenna are lost, since comparison of signals is made after their detection.

4.3. Unmodulated Incoherent Interference Created from Two Points

By unmodulated incoherent interference is understood signals created by two interference sources $Ц_1$ and $Ц_2$ spaced and not connected with respect to phase of radiated signals. It will be shown below that the antenna of the suppressed ASN system tracks a certain fictitious point O, located on segment $Ц_1$, $Ц_2$ (Fig. 4.7). Point O frequently is called the center of mass of the paired sources. With identical power of sources $Ц_1$ and $Ц_2$ the antenna of the ASN system tracks their geometric center.

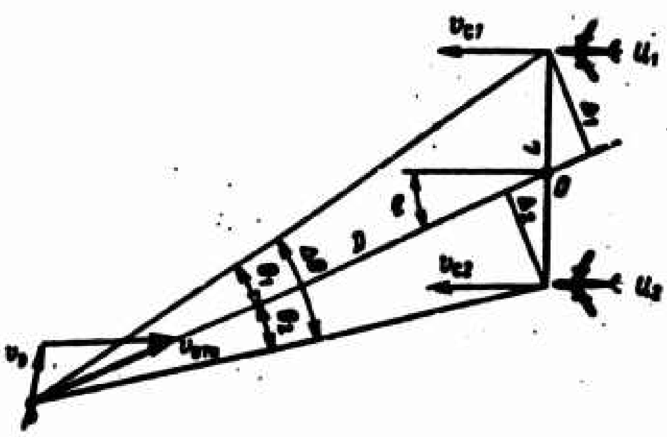

Fig. 4.7. Guidance of a guided missile to a paired target.

We will examine the influence of two signals arriving from different directions on the amplitude ASN system with simultaneous comparison of signals.

Let us designate angles between the equisignal direction of the suppressed system and first source of signals Ц$_1$ by θ_1, the second source Ц$_2$ by θ_2, and the angular distance between them by $\Delta\theta$ (Figs. 4.1 and 4.8).

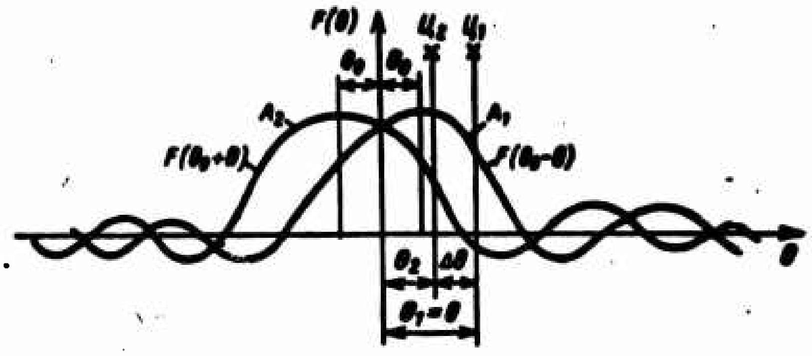

Fig. 4.8. Radiation patterns of antennas of ASN system.

The signals at the outputs of antennas A_1 and A_2 respectively will record in the form

$$u_1=U_1F(\theta_0-\theta_1)\cos\omega_1 t+U_2F(\theta_0-\theta_2)\cos\omega_2 t, \quad (4.14)$$

$$u_2=U_1F(\theta_0+\theta_1)\cos\omega_1 t+U_2F(\theta_0+\theta_2)\cos\omega_2 t, \quad (4.15)$$

where U_1 and ω_1 - amplitude and frequency of signal from first source Ц$_1$;

U_2 and ω_2 - amplitude and frequency of signal from second source Ц$_2$.

At the input of the sum channel we obtain

$$u_c=U_1[F(\theta_0-\theta_1)+F(\theta_0+\theta_1)]\cos\omega_1 t+$$
$$+U_2[F(\theta_0-\theta_2)+F(\theta_0+\theta_2)]\cos\omega_2 t.$$

At the input of the difference channel

$$u_p=U_1[F(\theta_0-\theta_1)-F(\theta_0+\theta_1)]\cos\omega_1 t+$$
$$+U_2[F(\theta_0-\theta_2)-F(\theta_0+\theta_2)]\cos\omega_2 t.$$

After conversion and amplification of signals in the i-f amplifier, voltages at the output of sum and difference channels are equal to

$$u_{c\,вых} = K_c\{U_1[F(\theta_0-\theta_1)+F(\theta_0+\theta_1)]\cos\omega_{пр1}t + U_2[F(\theta_0-\theta_2)+F(\theta_0+\theta_2)]\cos\omega_{пр2}t\}, \tag{4.16}$$

$$u_{p\,вых} = K_p\{U_1[F(\theta_0-\theta_1)-F(\theta_0+\theta_1)]\cos\omega_{пр1}t + U_2[F(\theta_0-\theta_2)-F(\theta_0+\theta_2)]\cos\omega_{пр2}t\}, \tag{4.17}$$

where K_c and K_p - gains of sum and difference channels; $\omega_{пр1}$, $\omega_{пр2}$ - intermediate frequencies of first and second signals.

At the output of the phase detector, which carries out the operation of multiplication and averaging of signals $u_{c\,вых}$ and $u_{p\,вых}$, we have

$$u_{фд} = \varkappa'\overline{u_{c\,вых}\,u_{p\,вых}}, \tag{4.18}$$

where $\varkappa'$ - is a constant.

Further, with the help of (4.16) and (4.17) we obtain

$$u_{фд} = \varkappa' K_c K_p\{U_1^2[F^2(\theta_0-\theta_1)-F^2(\theta_0+\theta_1)] + U_2^2[F^2(\theta_0-\theta_2)-F^2(\theta_0+\theta_2)]\}. \tag{4.19}$$

We will select as the start of reading of angles the direction to the first source $И_1$ (Fig. 4.8); then

$$\theta_1 = \theta,\ \theta_2 = \theta - \Delta\theta.$$

Designating

$$\beta = \frac{U_1}{U_2} \tag{4.20}$$

and taking into account (4.5), we obtain

$$u_{\phi д}=\kappa_{\phi д}\{\beta^2[F^2(\theta_0-\theta)-F^2(\theta_0+\theta)]+ +[F^2(\theta_0-\theta+\Delta\theta)-F^2(\theta_0+\theta-\Delta\theta)]\}, \tag{4.21}$$

where

$$\kappa_{\phi д}=\kappa' K^2 U_2^2.$$

Expression (4.21) determines the generalized direction finding characteristic, which constitutes dependence of voltage at output of phase detector on angular displacement of antenna θ to the first source Ц₁ [1]. In practice the generalized direction finding characteristic can be obtained if simultaneously with turn of the antenna of the inactive ASN system we record the voltage at the output of its phase detector while reading angular displacement of antenna (equisignal direction) from direction to first target $Ц_1$.

A family of generalized static characteristics $u_{\phi д}=u_{\phi д}(\theta)$, plotted for different angular distances $\Delta\theta$ = const between sources, most fully characterizes the behavior of the ASN system under the simultaneous influence of two incoherent signals. With the help of this family it is convenient to determine the position of points of stable equilibrium of the system for different angular distances between sources and relationships between powers β.

In Fig. 4.9 is given a family of generalized static characteristics, plotted in accordance with formula (4.21) under the assumption that $\kappa_{\phi д}=1$ and $\beta=1$ (powers of sources $Ц_1$ and $Ц_2$ are identical). In plotting the function describing the antenna radiation pattern was set up in the form

$$F(\theta)=e^{-1.4^2\left(\frac{\theta}{\theta_{0.5}}\right)^2}. \tag{4.22}$$

---

[1] The obtained expression (4.21) does not consider the influence of AGC on direction finding characteristics, which appears in dependence of gain of the i-f amplifier on characteristics of input signal. A more precise expression for the generalized direction finding characteristic will be obtained below [see formula (4.35)].

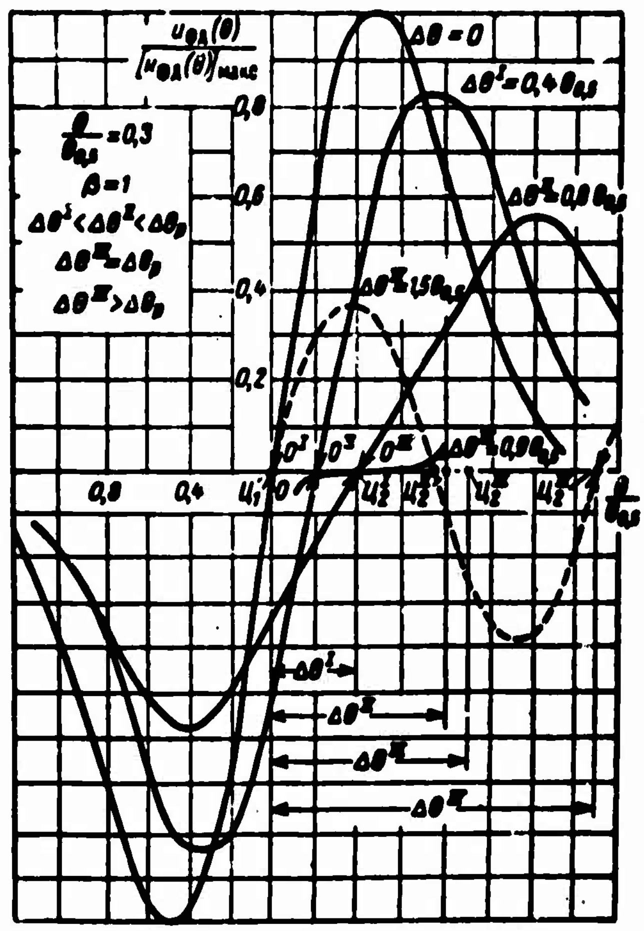

Fig. 4.9. Generalized direction finding characteristics during simultaneous action on ASN system of incoherent signals of two sources.

As can be seen from the figure, the direction finding characteristic is deformed with growth of angular distance Δθ between sources Ц_1 and Ц_2.

For equal amplitudes of interference signals U_1 and U_2 the system tracks the geometric center of the paired sources. This is attested to by the fact that in the process of increase of angular distance the point of stable state of equilibrium (0^{I}, 0^{II}) does not change position. It is exactly midway between directions to sources Ц_1 and Ц_2 ($\text{Ц}_1\text{Ц}_2^{I}$, $\text{Ц}_1\text{Ц}_2^{II}$).

However, with increase of angle $\Delta\theta$ the slope of the direction finding characteristic decreases (as does the transfer function of the system) at the point of stable equilibrium. With a certain critical angular distance an indifferent state of equilibrium is created in the system (transfer function of the system is equal to zero), and the antenna of the suppressed coordinator under the action of different random factors starts to be displaced from the geometric center of the paired target, approaching one of sources $Ц_1$ or $Ц_2$. This critical angle will be called the angle of resolution $\Delta\theta_p$ ($\Delta\theta_p = \Delta\theta^{III}$).

With further growth of angular distance $\Delta\theta$ ($\Delta\theta^{IV} >> \Delta\theta_p$) the center of the paired sources becomes the point of unstable equilibrium. Simultaneously there appear two points of stable equilibrium of the system, $Ц_1$ and $Ц_2^{IV}$, the positions of which are close to the true position of the sources. The antenna of the suppressed coordinator when $\Delta\theta > \Delta\theta_p$ begins to track one of paired sources $Ц_1$ or $Ц_2$. Complete resolution of targets prevails.

The qualitative analysis made of the behavior of the ASN system under the influence of interference from two points did not permit our estimating the value of the angle of resolution $\Delta\theta_p$. For a quantitative appraisal of angle $\Delta\theta_p$ we will use the condition characterizing stable equilibrium of the ASN system.

The geometric center of the paired sources will cease to be the point of stable equilibrium, and the ASN system will start to resolve targets $Ц_1$ and $Ц_2$, when the slope of the direction finding characteristic at this point is equal to zero. Mathematically the condition of resolution can be recorded in the following form

$$\left.\frac{\partial u_{\phi\Sigma}}{\partial\theta}\right|_{\theta=\frac{\Delta\theta}{2}} = 0. \qquad (4.23)$$

Using relationships (4.23) and (4.21), for approximation of the antenna radiation patterns (4.13) we obtain

$$\frac{\partial u_{\phi\Sigma}}{\partial\theta}=K_1\left\{\frac{1}{x_0-\frac{\Delta x}{2}}\left[F^2\left(x_0-\frac{\Delta x}{2}\right)-F(2x_0+\Delta x)\right]+\right.$$

$$\left.+\frac{1}{x_0+\frac{\Delta x}{2}}\left[F^2\left(x_0+\frac{\Delta x}{2}\right)-F(2x_0+\Delta x)\right]\right\}, \qquad (4.24)$$

where K_1 is a constant:

$$F(x)=\frac{\sin^2 x}{x};$$

$$x=\frac{\pi D}{\lambda}\sin\theta\approx\frac{\pi D}{\lambda}\theta;$$

$$x_0=\frac{\pi D}{\lambda}\sin\theta_0\approx\frac{\pi D}{\lambda}\theta_0;$$

$$\Delta x=\frac{\pi D}{\lambda}\sin\Delta\theta\approx\frac{\pi D}{\lambda}\Delta\theta.$$

The root of the transcendental equation $\frac{\partial u_{\phi\Sigma}}{\partial\theta}=0$ determines the angle of resolution by the coordinator of paired sources Ц₁ and Ц₂. Analysis of this equation shows that the angle of resolution of the paired sources is equal to

$$\Delta\theta_{p}=(0,8\div 0,9)\theta_{0.5}.$$

The value of the angle of resolution $\Delta\theta_p$ does not coincide with the value of the angular distance between peaks of the direction finding characteristic $\Delta\theta_{\text{макс}}$, plotted for the case of direction finding of a single source (Figs. 4.3 and 4.4).

Inequality of powers of paired sources Ц₁ and Ц₂ significantly affects the position of the axis of the equisignal zone of the suppressed ASN system. Let us determine the dependence of angular error tracking θ of the first source Ц₁ on the relationship between amplitudes of received signals. For this we linearize function $F(\theta)$ in the neighborhood of point $\theta = \theta_0$, corresponding to the equisignal direction. This is possible if the angular distance $\Delta\theta$ between sources

Ш₁ and Ш₂ is small as compared to θ_0. Taking into account the fact that

$$F(\theta_0 \pm \theta) = F(\theta_0) \mp |F'(\theta_0)|\theta,$$
$$F[\theta_0 \mp (\theta - \Delta\theta)] = F(\theta_0) \pm |F'(\theta_0)|(\theta - \Delta\theta),$$

we record expression (4.21) in the form

$$u_{\Phi Д} = 4\kappa_{\Phi Д} F'(\theta_0)[\theta(1+\beta^2) - \Delta\theta]. \qquad (4.25)$$

Since for the closed-loop ASN system

$$u_{\Phi Д} = 0,$$

from (4.25) we find

$$\theta = \frac{\Delta\theta}{1+\beta^2}. \qquad (4.26)$$

Hence it is clear that the equisignal direction will be oriented to the power center of the paired sources.

It is necessary to note that during the simultaneous influence on the ASN system of continuous signals (coherent or noncoherent) radiated from several points it is impossible to start with analysis of direction finding characteristics, plotted for the case of direction finding of a single target. The direction finding characteristic depicts nonlinear (with respect to angle) treatment of control signal by the ASN system, for which the principle of superposition is not preserved, which is the cause of deformation of direction finding characteristics plotted for the case of influence of two signals (Fig. 4.9).

In preceeding reasonings with respect to the influence of incoherent interference the power of both interference signals was assumed

identical. Let us now consider the influence of inequality of powers of paired sources Ц₁ and Ц₂ on the angular tracking error of the ASN system for one of sources Ц₁ or Ц₂. Let us determine angular error θ_1 in tracking of source Ц₁ (Fig. 4.10). For this we will use equation (4.21), the roots of which determine the position of the effective center of the paired sources.

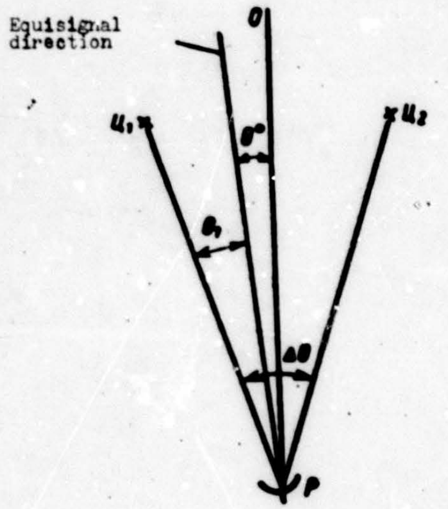

Fig. 4.10. Position of equisignal direction in the case of tracking of two sources of radiation.

On the basis of solution of this equation in Fig. 4.11 are plotted dependences of angle θ_1 on angular distance between targets $\Delta\theta$ for different relationships between amplitudes of signals β.

As follows from the figure, with increase of relationship β the error in tracking of the source with greater power decreases.

4.4. Influence of Noncoherent Interference on the Dynamics of ASN Systems.

Analysis of the family of direction finding characteristics (Fig. 4.9) shows that in process of closure between the ASN system and paired sources (increase of angle $\Delta\theta$) the slope of the direction

finding characteristic decreases at the point of stable equilibrium of the system. This slope, with accuracy of the constant factor, is equal to the transfer function of the direction finder (transfer factor of open ASN system), i.e.,

$$k_{\text{п у}} = \frac{\partial u_{\phi z}}{\partial \theta}\Bigg|_{\theta = \frac{\Delta\theta}{2}}. \tag{4.27}$$

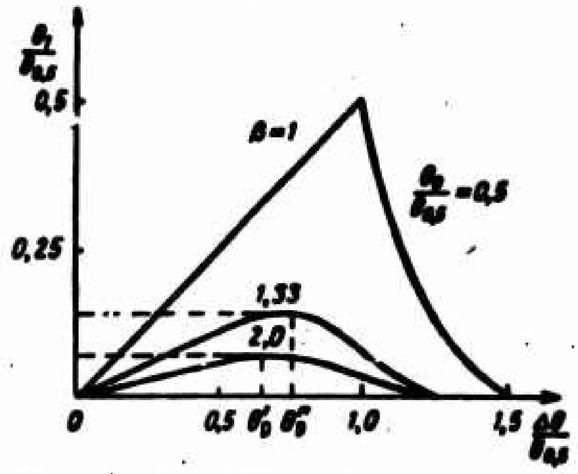

Fig. 4.11. Dependence of error in tracking one of the paired sources on angular distance between them.

Decrease of transfer function of the ASN system cannot fail to affect the quality of the transient process and dynamic error.

Let us determine the transfer function of the direction finder $k_{\text{п у}}$ during the influence on the ASN system of two incoherent signals. Noticeable influence on $k_{\text{п у}}$ is rendered, in addition to the value of angular distance between sources $\Delta\theta$, also by imperfectness of operation of the AGC. Allowance for the influence of the AGC can be made just as was done in finding the direction finding characteristic for the single source (4.2).

In the examined case at the input of the sum channel (Fig. 4.1)

we have

$$u_c = U \sin(\omega_1 t + \alpha), \tag{4.28}$$

where

$$U = \sqrt{U_{or_1}^2 + U_{or_2}^2 + 2U_{or_1} U_{or_2} \cos(\omega_2 - \omega_1) t}; \tag{4.29}$$

$$\mathrm{tg}\,\alpha = \frac{U_{or_1} + U_{or_2} \cos(\omega_2 - \omega_1) t}{U_{or_2} \sin(\omega_2 - \omega_1) t};$$

$$U_{or_1} = U_1 [F(\theta_0 - \theta) + F(\theta_0 + \theta)];$$

$$U_{or_2} = U_2 [F(\theta_0 - \theta + \Delta\theta) + F(\theta_0 + \theta - \Delta\theta)]. \tag{4.29a}$$

Expression (4.29) can be expanded in series

$$U = \sqrt{U_{or_1}^2 + U_{or_2}^2} + \frac{U_{or_1} U_{or_2}}{\sqrt{U_{or_1}^2 + U_{or_2}^2}} \cos \Omega_{\text{б}} t - \dots \tag{4.30}$$

Here

$$\Omega_{\text{б}} = \omega_2 - \omega_1.$$

In relationship (4.30) we will limit ourselves to two members of the series, since even in the most unfavorable case, when $U_1 = U_2$, the values of subsequent members are approximately one order of magnitude less. Let us record voltage U in the form

$$U \approx U_0 + U_{\text{б}} \cos \Omega_{\text{б}} t,$$

where

$$U_0 = U_{or_2} \sqrt{1 + \beta_0^2}; \tag{4.31}$$

$$U_{\text{б}} = \frac{U_{or_2} \beta_0}{\sqrt{1 + \beta_0^2}};$$

$$\beta_0 = \beta \frac{F(\theta_0 - \theta) + F(\theta_0 + \theta)}{F(\theta_0 - \theta + \Delta\theta) + F(\theta_0 + \theta - \Delta\theta)},$$

$$\beta = \frac{U_1}{U_2}.$$

Usually the beat frequency $\Omega_{\text{б}}$ is high in (kilohertz) as compared to the passband of the filter of the AGC feedback circuit (the bandwidth of the AGC filter, as a rule, does not exceed tens of hertz). Therefore it can be considered that at the output of the filter of the AGC feedback circuit there will be a d-c component of signal U_0 (with respect to frequency $\Omega_{\text{б}}$ the AGC system is open). Thus the control

voltage on grids of the tubes in steady-state operating conditions will be equal to

$$U_{рег} = K \varkappa_2 U_0, \tag{4.32}$$

where K – gain of sum channel;

$\varkappa_2$ – transfer function of detector and AGC amplifier.

In accordance with the adopted approximation (3.5) the gain of the sum channel is expressed by relationship

$$K = K_0 - \alpha U_{рег}. \tag{4.33}$$

Solving equations (4.32) and (4.33) jointly for K, we obtain

$$K = \frac{K_0}{1 + \alpha \varkappa_2 U_0},$$

or, considering (4.31) and (4.29a),

$$K = \frac{K_0}{1 + \mu'_2 \sqrt{1 + \beta_0^2}}, \tag{4.34}$$

where

$$\mu'_2 = \mu_2 [F(\theta_0 - \theta + \Delta\theta) + F(\theta_0 + \theta - \Delta\theta)];$$
$$\mu_2 = \alpha \varkappa_2 U_2.$$

Now it is possible to find the voltage at the output of the phase detector. Substituting (4.34) in (4.21), we finally obtain

$$u_{ф\,д} = \varkappa_{ф\,д} (\mu'_2)^2 \left\{ \frac{\beta^2 [F^2(\theta_0 - \theta) - F^2(\theta_0 + \theta)]}{(1 + \mu'_2 \sqrt{1 + \beta_0^2})^2} + \right.$$
$$\left. + \frac{F^2(\theta_0 - \theta \pm \Delta\theta) - F^2(\theta_0 + \theta - \Delta\theta)}{(1 + \mu'_2 \sqrt{1 + \beta_0^2})^2} \right\}. \tag{4.35}$$

Expression (4.35) determines the generalized direction finding characteristic of the ASN system for the case of direction finding of paired sources, with allowance for AGC.

Let us note that from formula (4.35) for $U_1 = 0$ as a particular case we obtain expression (4.10) for the direction finding characteristic during the direction finding of a single target.

Using relationship (4.27), from (4.35) we find the transfer function of the direction finder $\kappa_{\text{в у}}$ for the case of direction finding of paired sources of equal power $(\beta=1)$ for the adopted approximation of antenna radiation pattern (4.13)

$$\kappa_{\text{н у}} = \kappa_{\text{д}}\mu^{2}\left[\frac{\frac{1}{x_0-\frac{\Delta x}{2}}\left[F^{2}\left(x_0-\frac{\Delta x}{2}\right)-F(2x_0-\Delta x)\right]}{\left\{1+\frac{2\mu}{\sqrt{2}}\left[F\left(x_0-\frac{\Delta x}{2}\right)+F\left(x_0+\frac{\Delta x}{2}\right)\right]\right\}^{2}}+\right.$$
$$\left.+\frac{1}{x_0+\frac{\Delta x}{2}}\,\frac{F^{2}\left(x_0+\frac{\Delta x}{2}\right)-F(2x_0+\Delta x)}{\left\{1+\frac{2\mu}{\sqrt{2}}\left[F\left(x_0-\frac{\Delta x}{2}\right)+F\left(x_0+\frac{\Delta x}{2}\right)\right]\right\}^{2}}\right], \qquad (4.36)$$

where

$$F(x)=\frac{\sin x}{x};\ \mu=a\kappa_{0}U;\ U=U_1=U_2.$$

Curves of dependence of transfer function of direction finder on relative angular distance between sources $\Delta\theta/\theta_{0.5}$ for various μ and $\theta_0/\theta_{0.5} \approx 0.3$ are shown in Fig. 4.12. All curves are standarized relative to maximum transfer function $\kappa_{\text{в у макс}}$ with ideal AGC $(\mu=\infty)$. Investigation of the given curves shows that with increase of angular distance $\Delta\theta$ the transfer function of the direction finder $\kappa_{\text{в у}}$ decreases. When $\Delta\theta \approx 0.85\theta_{0.5}$ the system is in an indifferent state $(\kappa_{\text{в у}}=0)$, i.e., resolution of targets occurs.

The power of received signals (μ^{2}) does not affect the value of

the angle of resolution. With growth of signal power only the transfer function of the system is increased, where this increase continues to a defined limit, and for signals whose power corresponds to $\mu > 20$ one may assume that the transfer function $\kappa_{ву}$ is equal to the transfer function of the system with ideal AGC.

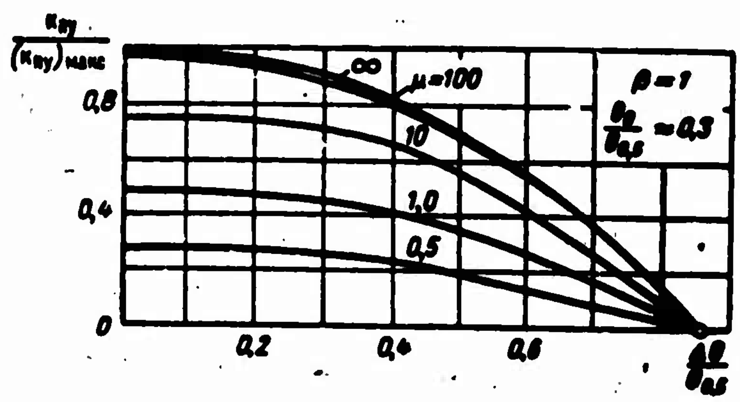

Fig. 4.12. Dependence of transfer function of open ASN system on angular distance between paired sources.

Decrease of the transfer function of ASN with astaticism of first order leads to growth of dynamic error. Thus if the input signal is changed per linear law

$$\theta_{вх} = \kappa_{вх} t,$$

where $\kappa_{вх}$ is the rate of change of input angle, the output signal will have an error in speed, which can be determined by the formula

$$\theta_{д} = \kappa_{д} \frac{\kappa_{вх}}{\kappa_{ву}(\Delta\theta)}, \tag{4.37}$$

where $\kappa_{д}$ is a constant.

It is necessary to note that formula (4.37) is valid for stationary or quasi-stationary systems and can be used as long as parameters of the system change relatively slowly.

4.5. Effectiveness of Noncoherent Interference

Let us examine the problem of guiding a missile **P** to a paired target $Ц_1$ and $Ц_2$ (Fig. 4.7) at attitude q.

In the beginning of guidance the distance to the paired target D considerably exceeds the value of interval L between targets $Ц_1$ and $Ц_2$ ($D \gg L$), and the angular distance between targets $\Delta\theta$ will be small. With equal effective scattering areas of both targets the goniometric coordinator of the homing device will track the geometric center 0 of the paired sources.

As the missile **P** approaches targets $Ц_1$ and $Ц_2$ the angle between sources $\Delta\theta$ increases and at a certain distance the missile's direction finder will distinguish both sources and will start to track one of them. The critical angle $\Delta\theta$, at which resolution of the paired sources occurs, is called the angle of resolution $\Delta\theta_p$.

We will consider that after resolving the sources the missile with maximum overload is guided to one of targets $Ц_1$ or $Ц_2$ (locked on), selecting the initial miss. As a result the missile will pass at a certain distance a from the target. The resultant miss a in this case can be determined in the following way.

From Fig. 4.7 it is clear that at the moment of resolution, to which the given figure corresponds, there are initial misses Δ_1 and Δ_2 respectively for targets $Ц_1$ and $Ц_2$. If targets $Ц_1$ and $Ц_2$ have identical effective scattering areas,

$$\Delta_1 = \Delta_2 = \Delta_н \approx \frac{L}{2}\cos q, \qquad (4.38)$$

where L is base between paired sources $Ц_1$ and $Ц_2$.

The resultant miss

$$a = \Delta_н - \Delta_\phi. \qquad (4.39)$$

Let us consider that after the resolution the missile moves with maximum overload; therefore from (1.59)

$$\Delta_0 = \frac{1}{2} j_{\text{макс}} \frac{D^2}{v^2_{\text{отн}}}. \qquad (4.40)$$

If D >> L, then

$$D \approx \frac{L \cos q}{\Delta\theta_p}. \qquad (4.41)$$

Substituting (4.41) in (4.40), we obtain

$$\Delta_0 = \frac{1}{2} j_{\text{макс}} \frac{L^2 \cos^2 q}{\Delta\theta_p^2 v^2_{\text{отн}}}. \qquad (4.42)$$

Considering (4.38) and (4.42), from (4.39) we obtain the expression for the resultant miss

$$a = \frac{L}{2} \cos q - \frac{1}{2} j_{\text{макс}} \frac{L^2 \cos^2 q}{\Delta\theta_p^2 v^2_{\text{отн}}}. \qquad (4.43)$$

On Fig. 4.13 is depicted a graph of dependence of misses $\Delta_{\text{н}}$ and Δ_0 on the value of base distance L. From the graph it is clear that the resultant miss a has maximum value $(a = a_{\text{макс}})$ for a certain $L = L_{\text{опт}}$.

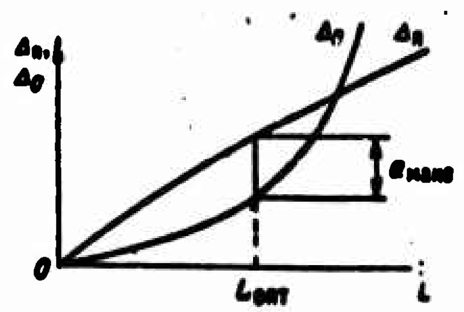

Fig. 4.13. Dependence of miss $(\Delta_{\text{н}})$, generated by the action of non-coherent interference and miss (Δ_0), selected during the time of homing guidance on base distance (L) between sources.

Let us find the optimum distance between sources $L_{\text{опт}}$. Differentiating (4.43) with respect to the variable L and equating the derivative to zero, we find

$$L_{\text{опт}}=\frac{1}{2}\,\frac{\Delta\theta_p^2\,v_{\text{отн}}^2}{j_{\text{макс}}\cos q}. \qquad (4.44)$$

Taking into account (4.44) and (4.43), we find the expression for maximum miss obtained through creation of noncoherent interference from two points in space:

$$a_{\text{макс}}=\frac{L}{4}\cos q$$

or

$$a_{\text{макс}}=\frac{\Delta\theta_p^2\,v_{\text{отн}}^2}{8j_{\text{макс}}}. \qquad (4.45)$$

Formula (4.45) shows that the miss depends in strong degree on the angle of resolution $\Delta\theta_p$ of the paired sources and on rate of closure between missile and target $v_{\text{отн}}$.

4.6. Interference Created by Two Coherent Sources of Radiation

As was shown earlier, in that case when two sources of signals are incoherent, the automatic tracking system follows the center "of mass" of the sources.

If the input of the antenna is acted upon by signals of two coherent sources, it is possible in principle to create of such a resultant signal by which the equisignal direction (RSN) of direction finder is oriented to a point located beyond the limits of the base [41, 43, 46-50]. Let us explain this in somewhat greater detail. In Fig. 4.14 are depicted the antenna array of an amplitude direction finder

with sum-difference treatment and its radiation pattern in a rectangular system of coordinates, taken respectively for radiators I and II.

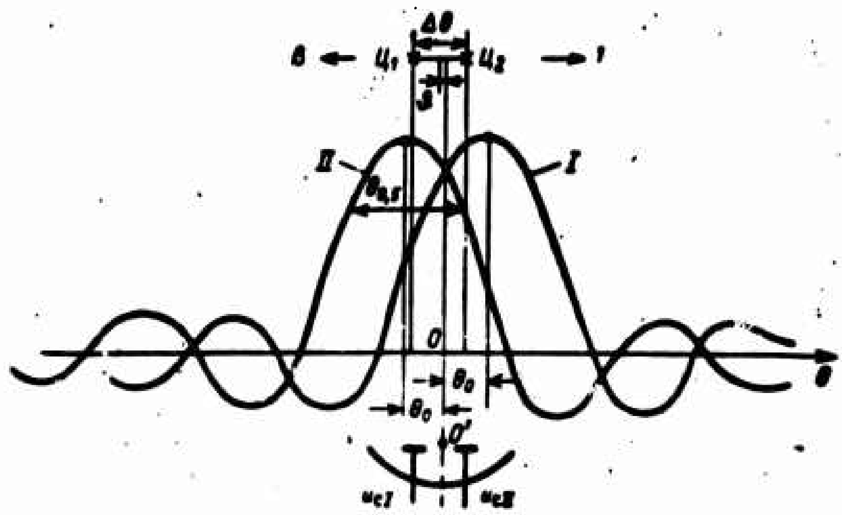

Fig. 4.14. Section of the radiation pattern of the antenna of a monopulse amplitude direction finder.

If bearings are being taken for a single point source, a zero signal at the output of the difference channel of the sum-difference circuit, and consequently that at the output of the phase detector, will be obtained in that case when the direction to the source of radiation coincides with equisignal direction 00'. In other words, zero voltage at the output of the phase detector at the point of stable equilibrium (of the direction finding characteristic) corresponds to the position on the axis of angles of equisignal direction, determined in accordance with both the principle beam of the radiation pattern and in terms of side lobes.

Let us determine the position of the RSN on the axis of angles for simultaneous influence on the direction finder of coherent signals of two spaced sources. We will be interested in the field of sources directly in the aperture of the direction finder antenna. If we ignore the stability of the direction finder, to the equisignal direction will correspond a zero signal at the output of the difference channel, i.e., the necessary condition determining the position of the equisignal direction on the axis of angles is zero voltage at the output of the

difference channel. Determination of sufficient conditions requires additional investigation of the stability of the ASN system in the neighborhood of the given zero point.

Let us assume that the sources of coherent signals are seen from a point corresponding to the geometric center 0' of the antenna of the suppressed direction finder at angle $\Delta\theta$ (Fig. 4.14).

The angle between the RSN of the direction finder and the direction to the middle of the base of sources of interference signals will be designated by ϑ.

Let us say that the amplitude of the field in the aperture of the antenna of the coordinator created by source $Ц_2$ is equal to 1 and that the amplitude of the field created by source $Ц_1$, is equal to β.

Let us find the resultant signal at the output of the difference channel of the sum-difference monopulse direction finder (coordinator). Let us consider two cases.

Case 1. Oscillations created by sources in the center of the aperture of the antenna of the coordinator are in phase ($\psi = 0$).

Let us find the total signal generated by sources $Ц_1$ and $Ц_2$, at outputs of radiators I and II respectively:

$$u_{cI} = \beta F\left(\theta_0 + \frac{\Delta\theta}{2} + \vartheta\right) + F\left(\theta_0 - \frac{\Delta\theta}{2} + \vartheta\right), \quad (4.46a)$$

$$u_{cII} = \beta F\left(\theta_0 - \frac{\Delta\theta}{2} - \vartheta\right) + F\left(\theta_0 + \frac{\Delta\theta}{2} - \vartheta\right). \quad (4.46b)$$

Here $F_i(\theta)_{i=1}$ represents the radiation pattern of the antenna. Angle θ is measured, as usual, from the direction of the radiation peak. The problem at hand (determination of position of the RSN on the axis of angles) can be solved strictly by representing radiation pattern $F(\theta)$ in the form of $F(x) = \frac{\sin x}{x}$ (antenna has a rectangular aperture with

side d, $x=\frac{\pi d}{\lambda}\sin\theta$) or $F(x)=\frac{2J_1(x)}{x}$ (outlet of antenna - aperture - constitutes a circle with diamter d, $x=\frac{\pi d}{\lambda}\sin\theta$), if we graphically find the roots of the transcendental equation

$$u_{cI}-u_{cII}=0. \tag{4.47}$$

Will limit ourselves to approximate consideration, for which we linearize function $F(\theta)$ in the neighborhood of the equisignal zone (RSZ). This can be done if $\Delta\theta/2$ and θ are small values as compared to θ_0, then

$$\begin{aligned} u_{cI}&=\beta\left[F(\theta_0)-|F'(\theta_0)|\left(\frac{\Delta\theta}{2}+\theta\right)\right]+\\ &\quad+F(\theta_0)+|F'(\theta_0)|\left(\frac{\Delta\theta}{2}-\theta\right),\\ u_{cII}&=\beta\left[F(\theta_0)+|F'(\theta_0)|\left(\frac{\Delta\theta}{2}+\theta\right)\right]+\\ &\quad+F(\theta_0)+|F'(\theta_0)|\left(\frac{\Delta\theta}{2}-\theta\right), \end{aligned} \tag{4.48}$$

$$u_{cI}-u_{cII}=-2\beta|F'(\theta_0)|\left(\frac{\Delta\theta}{2}+\theta\right)+2|F'(\theta_0)|\left(\frac{\Delta\theta}{2}-\theta\right). \tag{4.49}$$

From condition $u_{cI}-u_{cII}=0$ we find

$$\theta=\frac{\Delta\theta}{2}\,\frac{1-\beta}{1+\beta}. \tag{4.50}$$

The last expression shows that the equisignal zone of the ASN system in this case will be directed to a certain point between sources of interference signals (amplitude center "of mass").

When $\beta=1$ the axis of the RSZ is oriented to the middle of the base of interference signals.

When $\beta\to\infty$ and $\beta\to 0$ the axis of the RSZ is oriented to sources Ш_1 and Ш_2.

Case 2. Fields generated by coherent sources in the center of the aperture of the coordinator antenna (direction finder) are in opposite phase ($\psi = \pi$).

In this case

$$
\begin{aligned}
u_{eI} = \beta\left[F(\theta_0) - |F'(\theta_0)|\left(\frac{\Delta\theta}{2} + \theta\right)\right] - F(\theta_0) - \\
- |F'(\theta_0)|\left(\frac{\Delta\theta}{2} - \theta\right), \\
u_{eII} = \beta\left[F(\theta_0) + |F'(\theta_0)|\left(\frac{\Delta\theta}{2} + \theta\right)\right] - \\
- F(\theta_0) + |F'(\theta_0)|\left(\frac{\Delta\theta}{2} - \theta\right),
\end{aligned}
\tag{4.51}
$$

$$
u_{eI} - u_{eII} = -2\beta|F'(\theta_0)|\left(\frac{\Delta\theta}{2} + \theta\right) - 2|F'(\theta_0)|\left(\frac{\Delta\theta}{2} - \theta\right).
\tag{4.52}
$$

From condition $u_{eI} - u_{eII} = 0$ we find

$$
\theta = \frac{\Delta\theta}{2}\,\frac{1+\beta}{1-\beta}.
\tag{4.53}
$$

In that case when the phase difference of oscillations created by interference signals in the center of the antenna aperture is equal to ψ, we obtain the following expression for the angle between the RSN and the middle of base L of sources of interference signals:

$$
\theta = \frac{\Delta\theta}{2}\,\frac{1-\beta^2}{1+2\beta\cos\psi+\beta^2}.
\tag{4.54}
$$

From this formula relationships (4.50) and (4.53) are obtained as particular cases for $\psi = 0$ and $\psi = \pi$.[1]

[1]Formula (4.54) was obtained by B. I. Polikarov as a result of analysis of the joint influence of two coherent signals on the open-loop tracking system.

It is necessary to note that the formulas given above are approximate, inasmuch as they were obtained by way of linearization of the function describing the radiation pattern of the antenna.

Formulas (4.53) and (4.54) can be used only for small values of angles ϑ, satisfying the approximate equality tg $\vartheta \approx \vartheta$.

When β is close to unit (4.53) gives erroneous results. The value of anglular error ϑ in this case must be found by solving transcendental equation (4.47). More exactly, the limits of applicapability of formula (4.54) can be determined by the following inequalities: $\Delta\theta/\theta_{0,5} \leqslant 0{,}02+0{,}04$; $\beta \leqslant 0{,}9$ (or $\beta \geqslant 1{,}1$); here $\theta_{0.5}$ is width of beam of radar antenna.

As was noted earlier, the simultaneous influence on the amplitude direction finder of two coherent signals of opposite phase leads to angular displacement of equisignal direction beyond the base of the interference sources. The biggest value of angular displacement exists (neglecting the desired signal) during equality of amplitudes of interference signals.

Let us estimate approximately the value of angular error, using for this antenna radiation patterns depicted in Fig. 4.14. It is obvious that with equality of amplitudes of reversed phase interference signals, owing to opposition of signs of slope of radiation patterns I and II in the region of angles $-\theta_0$, $+\theta_0$ the difference $u_{cI}-u_{cII}$ does not turn to zero not for any value of ϑ, satisfying inequality $-\theta_0<\vartheta<\theta_0$. Equality (4.47) will apply only in the range of angles ϑ, in which the signs of the slope of radiation patterns are identical. These regions lie to the right and left of internal $-\theta_0$, $+\theta_0$, i.e., on the right and left slopes of radiation patterns I and II. Numerical solution of equation (4.47) shows that the maximum value of angular displacement of the RSN, on the average, is $0.6\theta_{0.5}$.

Let us explain the principle of action of coherent interference on the basis of analysis of the fine structure of the phase front of the resultant electrical vector of the field of two sources.

Let us assume that there are two coherent cophasal point sources Ц_1 and Ц_2 with amplitudes equal to β and 1 respectively. We investigate the space phase characteristic of the resultant electrical vector of the field at a distance much greater than the length of the base between sources L (zone of Fraunhofer diffraction). We will use a spherical system of coordinates for this purpose.

If there is one source of radiation, the line of equal phases constitutes a sphere whose center coincides with the source of radiation. The phase characteristic of two sources of radiation of identical amplitude also constitutes a sphere, but its center is at the median point of base L of the sources, in the so-called electrical (phase) center of the sources. (By definition the electrical center of several sources is understood as a point about which the phase characteristic constitutes a sphere). The direction of the normal to the phase front of a wave created one source coincides with the direction to this source.

The case of two coherent sources of equal amplitude the direction of the normal to the phase front coincides with the direction to the center of base L, if the fields of the sources are in phase in the center of the antenna aperture.

Let us examine the phase characteristic of two cophasal sources of equal amplitude more carefully, assuming that L >> λ. The amplitude characteristic of the total electrical vector of two sources (antenna radiation pattern is described by function

$$u_\Sigma = \cos \frac{\pi L}{\lambda} \sin \theta.$$

The phase characteristic constitutes a sphere (ϕ_0 = const), but with transition from lobe to lobe a sudden phase shift of π takes place (Fig. 4.15).

If, however, the amplitudes of the sources are not equal, amplitude and phase characteristics are transformed appropriately (Fig. 4.16).

The radiation pattern of two radiators of different amplitude on the border between lobes passes smoothly from a certain value ϕ to $\varphi\pm\pi$ in the interval of angles $\Delta\theta'$ of finite width $(\Delta\theta'\neq 0)$. In the region of angles $\Delta\theta'$, corresponding to a jump or smooth change of phase by $\pm\pi$ (region of phase inversion), the amplitude characteristic has minimum value. In Fig. 4.17 in a Cartesian system of coordinates is shown one of the sections of amplitude and phase characteristics. To the point of greatest slope of the phase characteristic $\phi(\theta)$ corresponds the least value of amplitude of the summed signal.

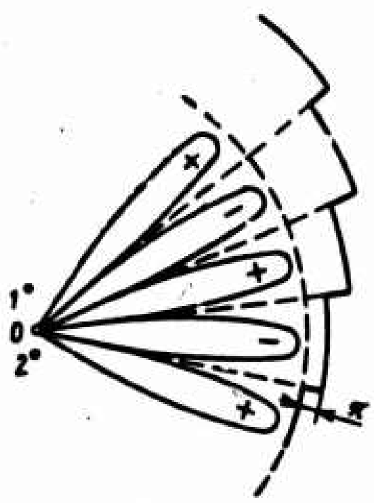

Fig. 4.15. Amplitude and phase characteristics of two coherent sources with equal power.

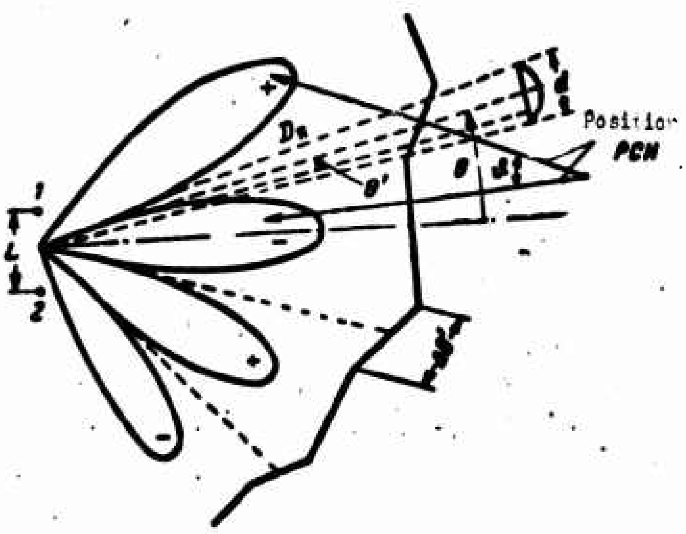

Fig. 4.16. Effect of coherent interference on ASN system.

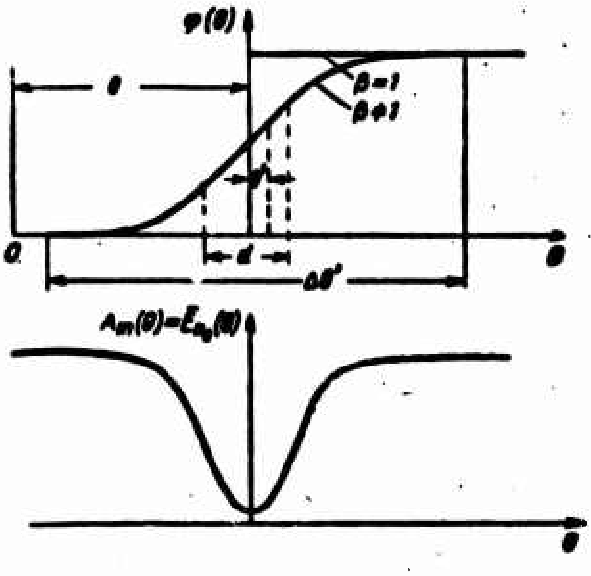

Fig. 4.17. Phase and amplitude characteristics of two reversed phase coherent sources of different power.

At sufficiently great distances from the sources of radiation the linear dimensions of the region of phase inversion can considerably exceed the dimensions of the direction finder antenna, which makes it possible to consider the section of the wavefront with phase and amplitude inversion within the limits of the antenna aperture flat. Because of the principle of action of the automatic direction finder, the axis of the antenna, coinciding with the RSN, is automatically oriented perpendicular to this section of the front with phase inversion. Accordingly the direction finder will track the center of the paired sources with error ϑ (Fig. 4.16).

Let us estimate the value of angular error, applying the above-stated concept of the fine structure of the phase front.

It is obvious that angle ϑ between normals to the spherical section of the front and the section with phase inversion (angular tracking error) equals the angle between corresponding tangents to these surfaces (Fig. 4.18). Hence we will apply the following means of solution of the problem:

1) find the equation for the line of equal phases of the electrical vector of the resultant field of two sources in the plane determined by the interference sources and the antenna of the suppressed radar;

2) find angle ϑ between tangents to the circle with radius r_0 and center at point 0 and lines of equal phases of the resultant field of the two coherent sources.

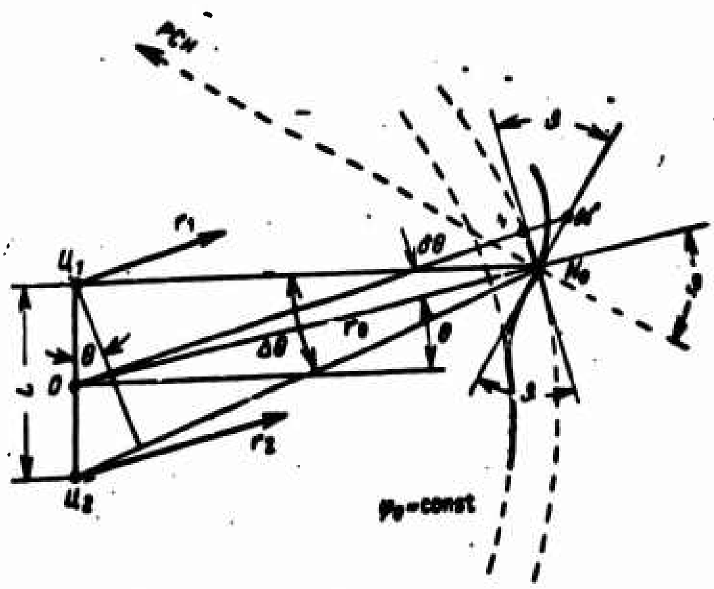

Fig. 4.18. Line of equal phases for the case of two coherent sources of different amplitude.

The thus obtained value of angle ϑ determines the angular tracking error of the direction finder if its antenna has linear dimension much smaller than the extent of the region of phase inversion.

The elementary vector of the resultant field of two coherent cophasal[1] sources at point N_0 at distance $r_0 \gg L$ can be recorded in the following way:

$$E_{N_0} = E_1 + E_2 = \frac{\kappa_0 E_{02}}{r_0}(\beta e^{jkr_1} + e^{jkr_2}) e^{-j\omega t}. \tag{4.55}$$

Here

---

[1]The assumption of the cophasal nature of fields is not fundamental, inasmuch as the discussion concerns the phase characteristic in space, but such assumption considerably simplifies the derivation.

$$\beta=\frac{E_{02}}{E_{01}};$$
$$k=\frac{2\pi}{\lambda};$$

E_{01} and E_{02} – amplitudes of fields in apertures of antennas of interference sources;

κ_0 – proportionality factor;

r_1 and r_2 – distances from sources to point of observation N_0;

ω – angular frequency.

Inasmuch as $r_0 \gg L$, with sufficient accuracy

$$r_1=r_0-\frac{L}{2}\sin\theta,$$
$$r_2=r_0+\frac{L}{2}\sin\theta, \tag{4.56}$$

Consequently

$$E_{N_0}=\frac{\kappa_0 E_{01}}{r_0}\left\{\beta\left[\cos\left(\frac{\pi L}{\lambda}\sin\theta\right)-j\sin\left(\frac{\pi L}{\lambda}\sin\theta\right)\right]+\right.$$
$$\left.+\cos\left(\frac{\pi L}{\lambda}\sin\theta\right)+j\sin\left(\frac{\pi L}{\lambda}\sin\theta\right)\right\}e^{-j\omega t} \tag{4.57}$$

or

$$E_{N_0}=\frac{\kappa_0 E_{01}}{r_0}\left[(1+\beta)\cos\left(\frac{\pi L}{\lambda}\sin\theta\right)+\right.$$
$$\left.+j(1-\beta)\sin\left(\frac{\pi L}{\lambda}\sin\theta\right)\right]e^{-j\omega t}. \tag{4.58}$$

Hence amplitude and phase characteristics of the electrical vector of the resultant field are obtained directly.

Amplitude Characteristic

$$E_{N_0}(\theta)=\sqrt{(1+\beta)^2\cos^2\left(\frac{\pi L}{\lambda}\sin\theta\right)+(1-\beta)^2\sin^2\left(\frac{\pi L}{\lambda}\sin\theta\right)}$$

or

$$E_{N_0}(\theta)=\sqrt{\beta^2+2\beta\cos\left(\frac{2\pi}{\lambda}L\sin\theta\right)+1}. \quad (4.59)$$

We will designate by ψ the phase difference of oscillations from source И_1 and И_2 at point N_0 (in the center of the aperture of the antenna of the suppressed radar). It is obvious that $\psi=\frac{2\pi}{\lambda}$ L sin θ. When $\beta=1$ from (4.59) we obtained the well-known formula for the amplitude characteristic

$$E_{N_0}(\theta)=\cos\left(\frac{\pi L}{\lambda}\sin\theta\right).$$

Phase Characteristic

$$\varphi(\theta)=\mathrm{arctg}\left[\frac{1-\beta}{1+\beta}\,\mathrm{tg}\left(\frac{\pi L}{\lambda}\sin\theta\right)\right]. \quad (4.60)$$

With the help of (4.60) we plotted the phase characteristic in Fig. 4.17. We will change angle θ by $\delta\theta$ and estimate the phase shift $\delta\varphi$ generated by this change.

In accordance with the definition of phase front

$$\delta\varphi=\delta r\frac{2\pi}{\lambda},$$

where

$$\delta r=NN'.$$

From triangle N_0NN' (Fig. 4.18) it follows that with accuracy of infinitesimal values of second order

$$\operatorname{tg}\vartheta = \frac{NN'}{N_0N}. \tag{4.61}$$

Here

$$N_0N = 2\delta\theta.$$

Hence

$$\operatorname{tg}\vartheta = \frac{\lambda}{2\pi r_0}\frac{\delta\varphi}{\delta\theta}$$

or, passing to the limit, we obtain

$$\operatorname{tg}\vartheta = \frac{\lambda}{2\pi r_0}\frac{d\varphi}{d\theta}. \tag{4.62}$$

Differentiating (4.60) with respect to θ, after making certain transpositions, we find

$$\operatorname{tg}\vartheta = \frac{L\cos\theta}{2r_0}\cdot\frac{1-\beta^2}{\beta^2+2\beta\cos\psi+1}. \tag{4.63}$$

From Fig. 4.18 it follows that $L\cos\theta/r_0=\Delta\theta$, i.e., the angle at which interference sources are seen from the center of the radar's antenna system. Considering this circumstances, we obtain the final formula for angular tracking error

$$\vartheta \approx \operatorname{tg}\vartheta = \frac{\Delta\theta}{2}\frac{1-\beta^2}{\beta^2+2\beta\cos\psi+1}. \tag{4.64}$$

Formula (4.64) coincides in accuracy with (4.54), which confirms the correctness of the proposed interpretation of the principle of creation of coherent interference. Once again let us remember that formula (4.64) can be used only for small values of angle $\vartheta(\vartheta\approx\operatorname{tg}\vartheta)$. In Fig. 4.19 is plotted dependence $\vartheta=\vartheta(\psi)$ for different values of amplitude ratio β. The biggest values of ϑ exist when $\psi=\pi$ and $\beta\rightarrow1$.

A qualitative explanation of the principle of coherent interference can be made with the help of vector diagrams of the field generated by interference sources.

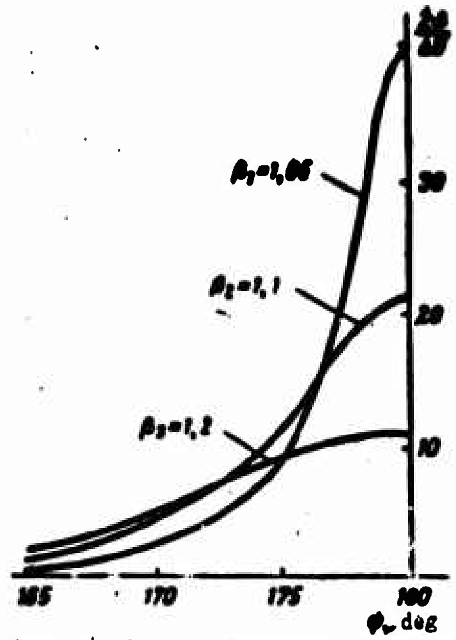

Fig. 4.19. Dependence of angular tracking error with respect to the geometric center of a pair of coherent sources on signal phase difference.

Electrical vectors $\bar{E}_1$ and $\bar{E}_2$ of sources are cophasal at the point of observation N_0 and can be represented as almost collinear. The angle between $\bar{E}_1$ and $\bar{E}_2$ is equal to angle $\Delta\theta$ (Fig. 4.20a). The Poynting vector $\bar{p}$ of the resultant field is perpendicular to the electrical vector of the resultant field E and lies on a straight line connecting point N_0 and a certain point between sources.

The equisignal direction of the direction finder, owing to its principles of action, is automatically oriented approximately along the Poynting vector.[1] Consequently at points where the fields of the interference sources are cophasal (in the center of the aperture of the antenna of the suppressed radar) the axis of the direction finder antenna is oriented to the amplitude (power) center of mass of these sources. If, however, the fields of the sources in the center of the antenna aperture are of opposite phase, the orientation of one of vectors $\bar{E}_1$ and $\bar{E}_2$ will be reversed accordingly (in Fig. 4.20b the orientation of vector $\bar{E}_1$ is changed). The Poynting vector is oriented

[1]The axis of the antenna is oriented exactly along the Poynting vector in the case of finding the direction of a point source.

along a straight line passing through point N_0 and a point lying beyond the base of the two sources. The axis of the direction finder antenna (RSN) will be oriented approximately the same.

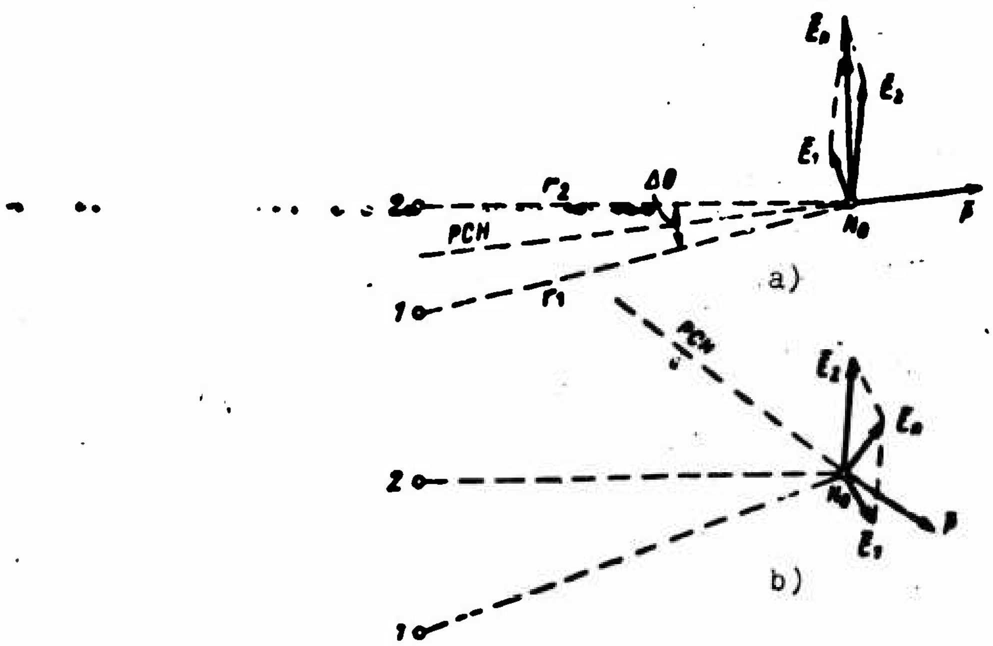

Fig. 4.20. Vector diagram of fields generated by two coherent sources: a) fields of sources of radiation 1 and 2 at point of observation N_0 are cophasal; b) fields of sources 1 and 2 at point of observation N_0 are of opposite phase.

Analysis of the amplitude-phase structure of space distribution of the resultant electrical vector (Fig. 4.17) suggests the idea of expanding the limits of applicability of formula (4.64) somewhat by way of averaging angles ϑ with respect to the aperture of the antenna of the suppressed radar (Fig. 4.16 and 4.17), taking into account the weight of each value of angle ϑ, determined by the corresponding value of amplitude of the resultant electrical vector of the field (4.59). It is most convenient as weighting function to select the square of amplitude. Then the average value of error can be determined in the following way:

$$\Delta\theta=\frac{\int\limits_{-\frac{d}{2D_n}}^{\frac{d}{2D_n}}\theta(\theta+\theta')P(\theta_0+\theta')d\theta'}{\int\limits_{-\frac{d}{2D_n}}^{\frac{d}{2D_n}}P(\theta+\theta')d\theta'},$$

where

$$P(\theta_0+\theta')=|E_{N_0}(\theta_0+\theta')|^2=E_2^2(\beta^2+2\beta\cos\varphi+1).$$

After substitution of value $\theta\approx\operatorname{tg}\theta$ from formula (4.64) we obtain the formula for the error generated by coherent sources

$$\bar{\theta}=\frac{\Delta\theta}{2}\,\frac{1-\beta^2}{\beta^2+2\beta\cos\frac{\sin x}{x}+1}. \qquad (4.65)$$

Here

$$x\approx\pi\frac{L\cos\theta_0}{D_n}\,\frac{d}{\lambda}\approx\pi\frac{\Delta\theta}{\theta_{0,5}}.$$

The limits of applicability of formula (4.65) can be determined only by comparing it with results of more strict solution or comparison with experiment. Its comparison was made with results obtained through graphical solution of transcendental equation (4.47). Results of comparison show that formula (4.64) is applicable for values of ratio $\Delta\theta/\theta_{0,5}\leqslant 0{,}05\div 0{,}1$ with values of $\beta\geqslant 1{,}1$ or $\beta\leqslant 0{,}9$.

For values β, close to unity formula (4.65), like formula (4.64), is inapplicable. More exactly, it becomes applicable with approach of β to unity for all smaller values of ratio $\Delta\theta/\theta_{0,5}$, and at the limit, when $\beta=1$ this ratio should be equal to zero. It is necessary to note that formula (4.64) gives error in the direction of overstated effectiveness, while formula (4.65) determines values of error smaller than actually exist (understates the effectiveness of jamming).

Until now we have examined action of only interference signals, while the desired signal was not considered.

Allowance for the useful signal can be made by different methods. It is most simply considered when interference and useful signal are noncoherent and the tracking system subjected to their simultaneous influence is in a state of equilibrium. Depending upon the operational mode of the second detector of the receiver, two cases can exist:

- the axis of the antenna (RSZ) is directed to the power center (square-law detector);
- the axis of the antenna (RSZ) is directed to the amplitude center (linear detector).[1]

We will limit ourselves to consideration of the case of reversed phase interference signals, these being the most interesting from the standpoint of jamming. Then in accordance with (4.53) the influence of interference signals can be replaced by the equivalent action of one source, located beyond the base at point 0' (Fig. 4.21), where source power

$$\bar{P}_{0\text{эк}} = \kappa_1 (E_{02} - E_{01})^2 = P_{02}(1-\beta)^2,$$

where κ_1 – proportionality factor;
P_{02} – radiated power of second interference source.

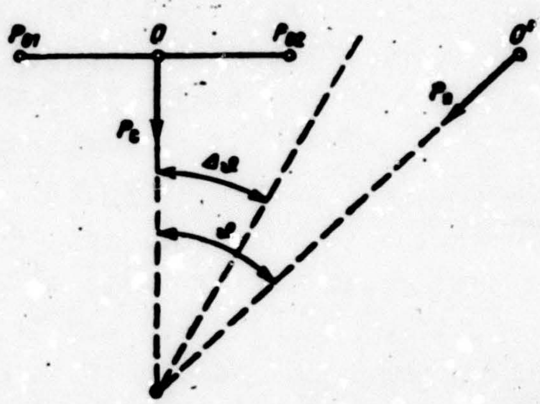

Fig. 4.21. Position of equisignal direction, with allowance for the influence of the desired signal.

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[1] Considered here are ASN systems with post-detector sum-difference treatment [60].

Assuming that the desired signal of power P_c is noncoherent with interference and is reflected from the center of the base 0, we record the condition of equilibrium of the closed loop tracking system in the case of the square-law detector and linear direction finding characteristic in the whole interval of examined angles (Fig. 4.21)

$$P_c \Delta\vartheta = P_\pi(\vartheta - \Delta\vartheta). \tag{4.66}$$

Here P_c and P_π are powers of useful and interference signals at output of antenna of suppressed radar.

Hence

$$\Delta\vartheta = \vartheta \frac{P_\pi}{P_\pi + P_c}. \tag{4.67}$$

Powers P_c and P_π can be expressed through powers of interference sources and power of suppressed radar:

$$P_\pi = \frac{P_{0\pi}(1-\beta)^2 G_\pi A_r \gamma_\pi}{4\pi D_\pi^2},$$

$$P_c = \frac{P_{0c} G_{0c} \sigma_\pi A_r}{4\pi D_\pi^2 4\pi D_\pi^2}.$$

Consequently,

$$k = \frac{P_\pi}{P_c} = (1-\beta)^2 \frac{P_{0\pi} G_\pi 4\pi D_\pi^2 \gamma_\pi}{P_{0c} G_{0c} \sigma_\pi}. \tag{4.68}$$

Thus,

$$\Delta\vartheta = \vartheta \frac{k}{1+k}. \tag{4.69}$$

The optimum ratio of amplitudes of interference sources for a given ratio of powers of interference source and suppressed radar can be obtained with the help of (4.69).

Let us replace in (4.69) ϑ and k by their values from (4.53) and (4.68), where we additionally introduce designation the $k = k'(1-\beta)$, where k' actually equals the ratio of power of interference source (in this case U_2) to power of desired signal at the input of the suppressed radar.

Thus for interference signals of opposite phase in the center of the aperture of the suppressed radar:

$$\Delta\vartheta = \frac{\Delta\theta}{2}\,\frac{k'(1-\beta^2)}{1+k'(1-\beta)^2} \tag{4.70}$$

or

$$\Delta\vartheta = \frac{\Delta\theta}{2}\,\frac{1-\beta^2}{(1-\beta)^2+\frac{1}{k'}}; \tag{4.71}$$

when $\beta \neq 1$ and $k' \gg 1$

$$\Delta\vartheta = \vartheta = \frac{\Delta\theta}{2}\,\frac{1+\beta}{1-\beta}.$$

If, however, $\beta = 1$, for finite values of k' we have $\Delta\vartheta = 0$.

The optimum value of β, ensuring the biggest value of $\Delta\vartheta$ for a given k', can now be found from (4.71). Differentiating (4.71) with respect to β and equating the expression to zero, we find the initial equation determining the optimum value of β:

$$\beta^2 - \left(2+\frac{1}{k'}\right)\beta + 1 = 0.$$

Hence

$$\beta_{\text{опт}} = 1 + \frac{1}{2k'} \pm \sqrt{\left(1+\frac{1}{2k'}\right)^2 - 1}. \tag{4.72}$$

With increase of power of sources of interference ($k' \to \infty$) the effectiveness of coherent interference determined by angular error $\Delta\vartheta$, grows, and the optimum value of the amplitude ratio of interference signals $\beta_{\text{опт}} \to 1$.

In the case of signals of greater amplitude (linear detection) the condition of equilibrium (4.66) is recorded in the following way:

$$\sqrt{P_c}\,\Delta\vartheta = \sqrt{P_п}\,(\vartheta - \Delta\vartheta). \qquad (4.73)$$

Accordingly

$$\Delta\vartheta = \vartheta \frac{\sqrt{k}}{1+\sqrt{k}}. \qquad (4.74)$$

Substituting values of ϑ and k from (4.53) and (4.68), we find

$$\Delta\vartheta = \frac{\Delta\theta}{2}\,\frac{(1+\beta)\sqrt{k}}{(1-\beta)\sqrt{k}+\sqrt{1-\beta}} \qquad (4.75)$$

or

$$\Delta\vartheta = \frac{\Delta\theta}{2}\,\frac{1+\beta}{1-\beta+\frac{\sqrt{1-\beta}}{\sqrt{k}}}. \qquad (4.76)$$

Till now we did not pay attention to the sign of angular error. In general, this question should be examined specially. In first approximation on the basis of the study of the fine structure of the phase front one may assume that the axis of the direction finder antenna will be directed beyond the base from the source of greater power. Owing to the symmetry of the system of interference sources, error can be either greater or also less than zero; therefore, in general the optimum value of β with increase of power of interference sources approaches unity on both the right and left, i.e., in the case of strong signals

$$\beta_{опт} \approx 1. \qquad (4.77)$$

The value of angular error corresponding to $\beta_{опт}$, should be determined by solving transcendental equation (4.47).

In many cases during reflections of electromagnetic waves from bodies of comparatively simple form part of the components of the desired signal will be coherent with the interference signals for prolonged intervals of time. This circumstance will cause a change of the phase relationships of resultant vectors of the field in the antenna aperture (Fig. 4.22). Instead of interference electrical vectors E_1 and E_2, shifted in phase by π, in the center of the antenna aperture will act vectors E_2 and E'_1, whose shift $\psi \neq \pi$. A slight change of phase relationship can lead to considerable change of angular error θ (Fig. 4.19).

Fig. 4.22. Phase relationships of electrical vectors of fields of useful and interference signals.

The formulas given above for determination of angular errors generated by "coherent" interferences do not consider fluctuations of phases and amplitudes of interference electrical vectors E_1 and E_2. Allowance for fluctuations can be made by way of corresponding averaging of θ or $\Delta\theta$.

In general, when the phase shift between interference signals in equal to ψ, total power of interference signal in the center of the aperture of the antenna of the suppressed radar is determined with help of formula (4.59)

$$P_{п} = P_{п2}(\beta^2 + 2\beta\cos\psi + 1), \qquad (4.78)$$

where $P_{п2}$ is power of interference signal of second source at the input of the radar.

With the help of (4.67) and (4.74) we will obtain formulas for

for angular error under the assumption of noncoherence of useful and interference signals:

– square-law detection

$$\Delta\theta = \frac{\Delta\theta}{2}\cdot\frac{1-\beta^2}{\beta^2+2\beta\cos\psi+1+\frac{1}{k}}; \qquad (4.79)$$

– linear detection

$$\Delta\theta = \frac{\Delta\theta}{2}\cdot\frac{1-\beta^2}{\beta^2+2\beta\cos\psi+1+\frac{1}{\sqrt{k}}+\sqrt{\beta^2+2\beta\cos\psi+1}}. \qquad (4.80)$$

Hence can be found optimum values of amplitude ratio of interference signals. In particular, with square-law detection

$$\beta_{\text{опт}} = -\frac{1+\frac{1}{2k}}{\cos\psi} \pm \sqrt{\frac{\left(1+\frac{1}{2k}\right)^2}{\cos^2\psi}-1}.$$

It is easy to see that the optimum value $\beta_{\text{опт}}>0$ exists if $\psi>\frac{\pi}{2}$.

4.7. Influence on the ASN System of Periodic Input Disturbance

It is assumed that the ASN system is acted upon by periodic input disturbance of form

$$\theta_{\text{вх}} = \theta^{\circ}_{\text{вх}} + A\sin\Omega t = \theta^{\circ}_{\text{вх}} + \theta^{*}_{\text{вх}}, \qquad (4.81)$$

where $\theta^{\circ}_{\text{вх}}$ – slowly changing component

A, Ω – amplitude and frequency of external influence.

Representation of input disturbance in form (4.81) corresponds to the case when the target or the object itself on which the ASN

system is carried goes through periodic oscillations (vibrations). If we assume that the frequency Ω of external disturbances is sufficiently high that it is outside the limits of the equivalent passband of the ASN system, $\theta^{0}_{вх}$ will characterize slow shifts of target.[1] Component $\theta^{0}_{вх}$ is the useful control signal and is processed by the ASN system. Component $\theta^{*}_{вх}$ is harmful, and its presence leads to a series of undesirable phenomena.

Naturally, representation of input signal in form (4.81) is an abstraction. In the general case $\theta_{вх}$ is a random function, but it sometimes is useful for rough estimates to assign input disturbance in the form of a harmonic. In particular, apparently, a qualitative appraisal of the influence of "wanderings" of the effective center of a complex target on the dynamics of the ASN system can be made if one assigns the harmonic input disturbance (4.81). Then the target can be considered a point, with its motion characterizing member $\theta^{0}_{вх}$. "Wanderings" of the effective center are assumed harmonic, with amplitude A and frequency Ω.

In an ASN system there usually are different nonlinear elements. Considerable nonlinearity is displayed by the direction finder, the AGC system, and others. The equivalent circuit of the tracking system of an ASN, with allowance for AGC, can be represented in the form of Fig. 4.23. Such representation of the ASN tracking system is quite general. The input signal of system is angle $\theta_{вх}$. The output quantity will be the voltage fed to the automatic pilot (or some other device) for formation of control signal in the homing guidance (guidance) circuit. The direction finder (measuring element) converts the geometric parameter (angle $\theta_{вх}$) to an electrical one $u'_{в}$. This element is nonlinear and is characterized by function

$$u'_{в} = F'(\theta).$$

[1]Subsequently it is considered that the frequency of the external disturbances is sufficiently high and that the ASN system is open with respect to this frequency Ω.

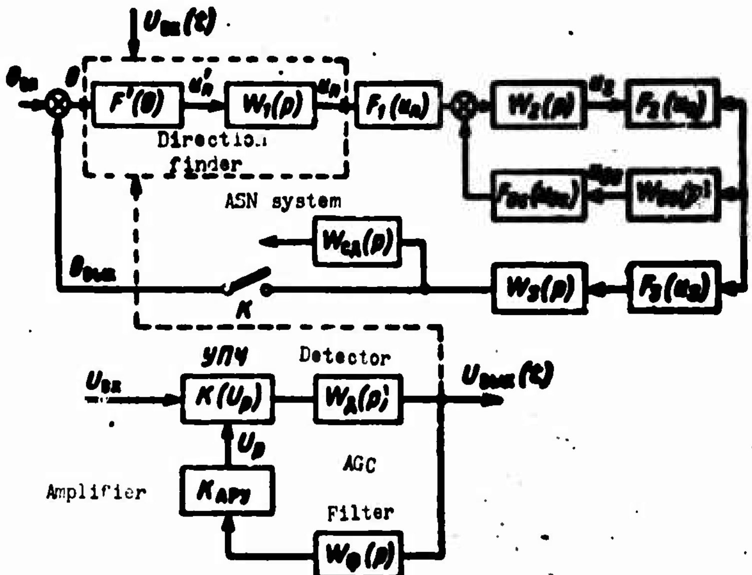

Fig. 4.23. Block diagram of ASN system.

The form of function $F'(\theta)$ is determined by the direction finding characteristic (Figs. 3.5, 3.6, 4.3, and 4.4). Transfer functions $W(p)$ characterize the following elements:

$W_1(p)$ – filter of phase detector and amplifiers of error signal;
$W_2(p)$ – amplifiers covered by feedback;
$W_3(p)$ – amplifiers and drive assembly;
$W_{o.c}(p)$ – feedback circuit;
$W_{c.д}(p)$ – data removal device.

The action of the AGC system appears as change of transfer function and form of nonlinearity $F'(\theta_{вх})$ of direction finder in dependence upon the amplitude of the input signal $U_{вх}(t)$. Subsequently it is considered that $U_{вх}=\mathrm{const}$, and influence of AGC is not considered.

The simplest and most effective means of study of nonlinear systems under the influence of harmonic disturbances is the method of harmonic linearization [100]. It is based on Fourier expansion of nonlinear oscillations at the output of the nonlinear element during the influence on its input of periodic disturbances. Here the nonlinear characteristic is replaced by a group of straight lines whose slope depends on amplitude A of oscillations of input variable $u(t)$. As a result the nonlinear element is replaced by a linear one with equivalent gain $K_э$.

the value of which depends, in general, on amplitude and frequency of input disturbance.

Basic nonlinearity of the ASN system is caused by the direction finder (measuring device).

Analysis of direction finding characteristics (Figs. 3.5, 3.6, 4.3 and 4.4) shows that with accuracy sufficient for practice the nonlinear function $F'(\theta)$ can be approximated by straight lines, shown in Fig. 4.24.

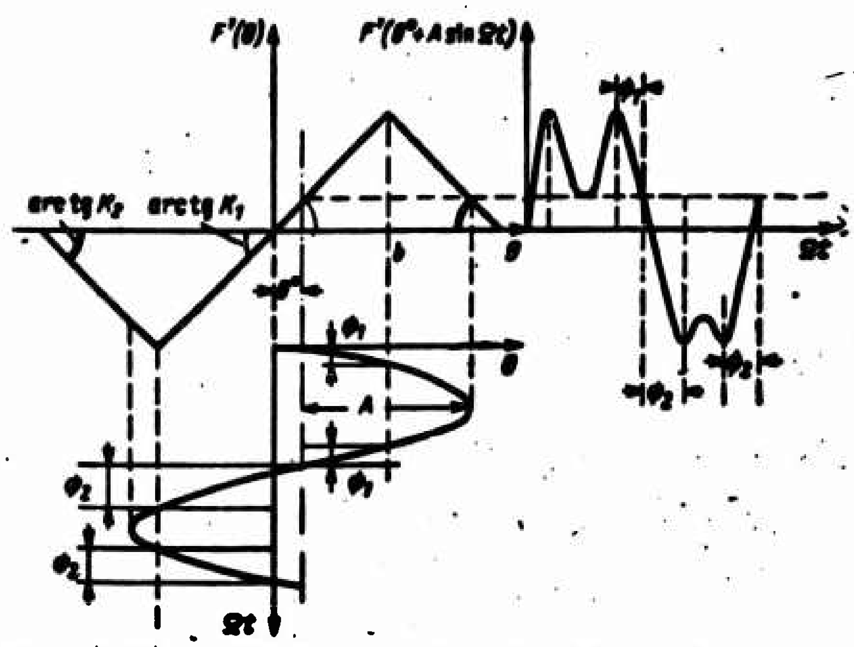

Fig. 4.24. Harmonic linearization of nonlinear direction finding characteristic under the action of external harmonic disturbance.

For the execution of harmonic linearization of nonlinear function $F'(\theta)$ for given asymmetric oscillations we will consider that the solution for the value of θ is sought in the form of (4.81). Then the nonlinear function $F'(\theta)$, on the basis of the method of harmonic linearization, is replaced by the following relationship:

$$F'(\theta) = F^{\circ} + q\theta^{\circ} = F^{\circ}(A, \theta^{\circ}) + q(A, \theta^{\circ})\theta^{\circ}, \qquad (4.82)$$

where F° and q are coefficients for the first two terms of expansion of $F'(\theta)$ in Fourier series:

$$F^{\circ}=\frac{1}{2\pi}\int_{0}^{2\pi}F'(\theta^{\circ}+A\sin\Omega t)\,d\Omega t; \tag{4.83}$$

$$q=\frac{1}{\pi A}\int_{0}^{2\pi}F'(\theta^{\circ}+A\sin\Omega t)\sin\Omega t\,d\Omega t. \tag{4.84}$$

For the constant component F° and coefficient q on the basis (4.83) and (4.84) and Fig. 4.24 we have

$$\begin{aligned}
F^{\circ}(A,\theta^{\circ}) &= -\kappa_2\theta^{\circ}+\frac{\kappa_1+\kappa_2}{\pi}\Big[A\Big(\sqrt{1-\frac{(b-\theta^{\circ})^2}{A^2}}-\\
&\quad-\sqrt{1-\frac{(b+\theta^{\circ})^2}{A^2}}\Big)+(b-\theta^{\circ})\arcsin\frac{b-\theta^{\circ}}{A}-\\
&\quad-(b+\theta^{\circ})\arcsin\frac{b+\theta^{\circ}}{A}\Big],\\
q(A,\theta^{\circ}) &= -\kappa_2+\frac{\kappa_1+\kappa_2}{\pi A}\Big[(b-\theta^{\circ})\sqrt{1-\frac{(b-\theta^{\circ})^2}{A^2}}+\\
&\quad+(b+\theta^{\circ})\sqrt{1-\frac{(b+\theta^{\circ})^2}{A^2}}+\\
&\quad+A\Big(\arcsin\frac{b-\theta^{\circ}}{A}+\arcsin\frac{b+\theta^{\circ}}{A}\Big)\Big].
\end{aligned} \tag{4.85}$$

where $A>b+|\theta^{\circ}|$.

Here $b=\frac{\Delta\theta_{\text{макс}}}{2}$ is half of the angular distance between maxima of the direction finding characteristic.

In ASN system considerable nonlinearity (after the nonlineary of the direction finder) is created also by saturation of error signal amplifiers in forward and feedback circuits. Since an increase in the linear section of the static amplitude characteristic of amplifiers is connected with sharp increase of weight and dimensions of amplifiers to avoid saturation in amplifiers during large values of mismatch signal is impossible. Therefore in order to get an objective picture of the influence of periodic disturbance it is necessary to consider also nonlinearities of saturation type $F_1(u_1)$, $F_2(u_2)$, $F_3(u_3)$, and $F_{o\ c}(u_{o\ c})$ in forward and feedback circuits (Fig. 4.25).

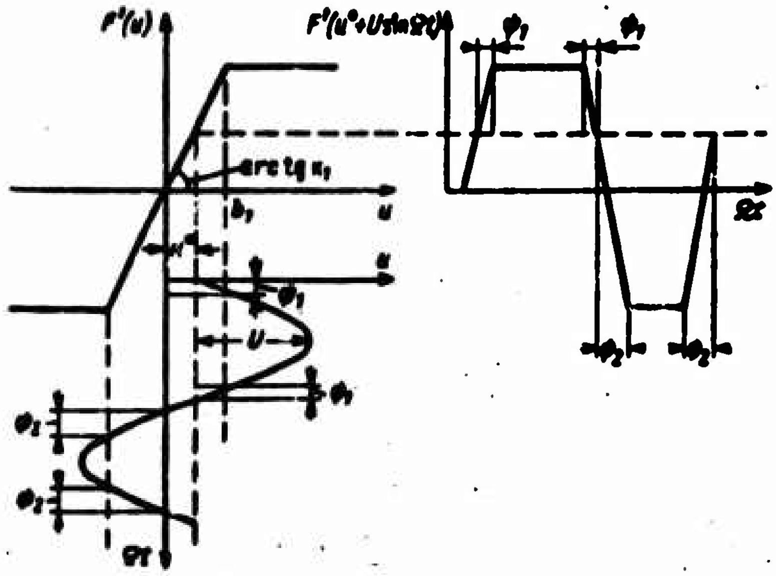

Fig. 4.25. Harmonic linearization of nonlinearity of saturation type under the action of external harmonic disturbance.

For nonlinearities of saturation type values of constant component F° and coefficient of harmonic linearization q can be recorded in the form

$$F^\circ(U,u^\circ)=\frac{k}{\pi}\left[U\left(\sqrt{1-\frac{(b_1+u^\circ)^2}{U^2}}-\sqrt{1-\frac{(b_1-u^\circ)^2}{U^2}}\right)+\right.$$
$$\left.+(b_1+u^\circ)\arcsin\frac{b_1+u^\circ}{U}-(b_1-u^\circ)\arcsin\frac{b_1-u^\circ}{U}\right], \qquad (4.86)$$
$$q(U,u^\circ)=\frac{k}{\pi}\left[\arcsin\frac{b_1-u^\circ}{U}+\arcsin\frac{b_1+u^\circ}{U}+\right.$$

$$\left.+\frac{b_1-u^\circ}{U}\sqrt{1-\frac{(b_1-u^\circ)^2}{U^2}}+\frac{b_1+u^\circ}{U}\sqrt{1-\frac{(b_1+u^\circ)^2}{U^2}}\right], \qquad (4.87)$$

where $U>b_1+|u^\circ|$.

Functions F° (4.85) and (4.86), frequently called displacement functions, are characteristics of corresponding nonlinear elements of system with respect to slowly changing component θ° (or u°). Function $F^\circ(A, \theta^\circ)$ is a smooth curve, close to linear in the neighborhood of point $\theta^\circ = 0$. This permits conducting usual linearization; namely, over a certain comparatively large section near the origin

of coordinates it is possible to take

$$F^{\circ} = K_{э} \theta^{\circ}, \tag{4.88}$$

where $K_{э}$ is equivalent gain of the linearized nonlinear element:

$$K_{э} = \left(\frac{\partial F^{\circ}}{\partial \theta^{\circ}} \right)_{\theta^{\circ} = 0}. \tag{4.89}$$

Then all slowly developing processes connected with change of angular position of target θ° can be described by a linear differential equation.

Since the system always tries to reduce mismatch θ° to zero, in practice relationship (4.88) will be valid for the entire item the ASN system is in a stable state of equilibrium.

The nonlinear element transfers the slowly changing component θ°, generated by target shift. Here it is very significant that gain depends not only on the structure and parameters of the ASN system itself but also on amplitude A and frequency Ω of the external influence.

On the basis of relationships (4.85), (4.86), and (4.86) can be obtained formulas for equivalent transfer function of direction finder $k_{п}$ and elements with saturation $k_{огр}$:

$$K_{п} = 2 \frac{K_1 + K_2}{\pi} \arcsin \frac{b}{A} - K_2, \tag{4.90}$$

$$K_{огр} = \frac{2K}{\pi} \arcsin \frac{b_1}{U}, \tag{4.91}$$

where $A > b$ and $U > b_1$.

Graphs of coefficients $K_{п}$ and $K_{огр}$ are shown in Figs. 4.26 and 4.27. From the graphs it follows that the gains of the nonlinear elements decrease with growth of amplitude of external influence

A and U. It is interesting that when amplitude A reaches a certain value $A = A_{кр}$ the transfer function of the direction finder even becomes negative, which indicates that the ASN system becomes unstable. In other words, external influence with amplitude $A > A_{кр}$ shifts the ASN system into an unstable state of equilibrium. More detailed investigations show that in beginning of influence, when $A = A_{кр}$ the ASN system goes into self-oscillation, and then with growth of A the amplitude of these oscillations is increased, and with further growth of A the ASN system can be driven out of the autotracking mode.

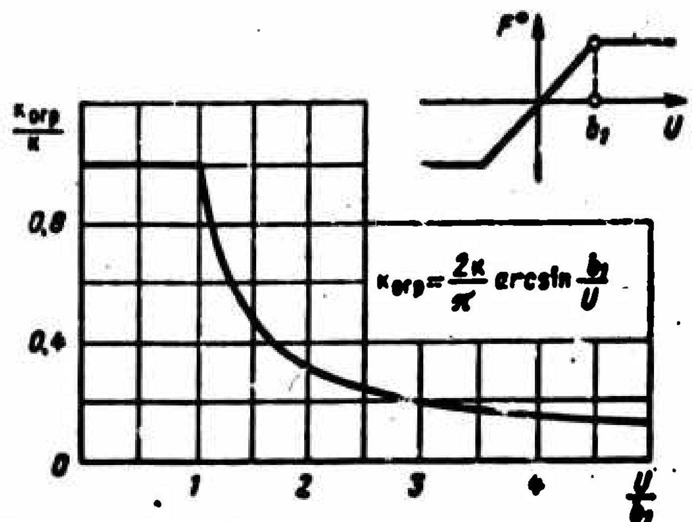

Fig. 4.26. Dependence of equivalent transfer function of direction finding characteristic on amplitude of external harmonic influence.

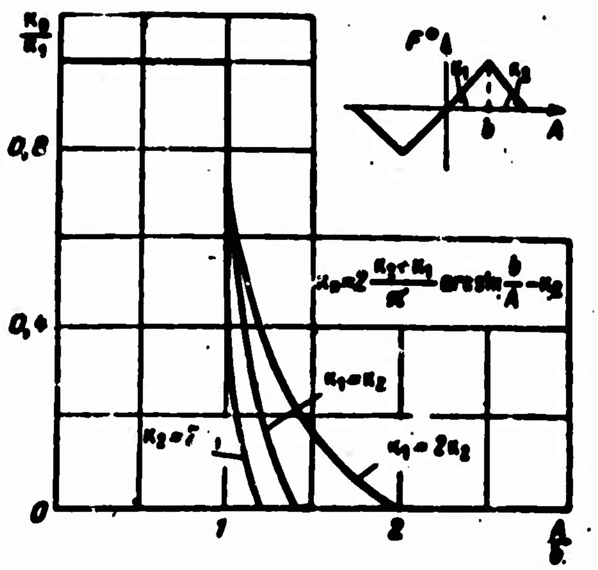

Fig. 4.27. Dependence of transfer function of amplifier with limitation on amplitude of external harmonic influence.

Solving equation

$$\frac{2(\kappa_1+\kappa_2)}{\pi}\arcsin\frac{b}{A}-\kappa_2=0 \tag{4.92}$$

gives us an expression for critical amplitude of external influence

$$\frac{A_{\text{кр}}}{b}=\operatorname{cosec}\frac{\pi\kappa_2}{2(\kappa_1+\kappa_2)} \tag{4.93}$$

or

$$A_{\text{кр}}=\frac{1}{2}\Delta\theta_{\text{макс}}\operatorname{cosec}\frac{\pi\kappa_2}{2(\kappa_1+\kappa_2)}. \tag{4.94}$$

when $\kappa_1>\kappa_2$

$$A_{\text{кр}}\approx 0.32\Delta\theta_{\text{макс}}\left(1+\frac{\kappa_1}{\kappa_2}\right). \tag{4.95}$$

For investigation of stability of a nonlinear system usually by breaking the circuit at certain point K we separate the nonlinear element from the remaining linear part (Fig. 4.28).

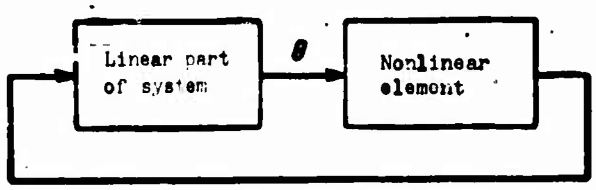

Fig. 4.28. Reduced block diagram of nonlinear ASN system for investigation of its stability.

The equation for the dynamics of the system in such case can be reduced in the form of the following differential equation [100]:

$$Q(p)\theta+R(p)F'(\theta)=S(p)\theta_{\text{вх}}(t). \tag{4.96}$$

where Q(p), R(p), and S(p) are polynomials of any degree with constant coefficients, where the degree of R(p) is lower than that of Q(p); F'(θ) is assigned nonlinearity;

$$\theta_{\text{вх}}=\theta^{\circ}_{\text{вх}}+\theta^{*}_{\text{вх}};$$

$\theta^{0}_{\text{вх}}$ – slowly changing external influence;
$\theta^{*}_{\text{вх}}$ – periodic external influence.

The ratio R(p)/Q(p) is the transfer function of the reduced linear part of the system, obtained way of breaking the system at a certain point.

Since for harmonic linearization

$$F(\theta) = F^{0} + q\theta^{*},$$

where

$$\theta = \theta^{0} + \theta^{*} = \theta^{0} + A\sin\Omega t,$$

that initial equation (4.96) can be recorded in the form

$$Q(p)(\theta^{0} + \theta^{*}) + R(p)(F^{0} + q\theta^{*}) = S(p)(\theta^{0}_{\text{вх}} + \theta^{*}_{\text{вх}}). \qquad (4.97)$$

The obtained equation for sufficiently slow change of function $\theta^{0}_{\text{вх}}(t)$ can be split into two, for slowly changing and oscillatory components respectively:

$$Q(p)\theta^{0} + R(p)F^{0} = S(p)\theta^{0}_{\text{вх}}, \qquad (4.98)$$

$$Q(p)\theta^{*} + R(p)q\theta^{*} = S(p)\theta^{*}_{\text{вх}}. \qquad (4.99)$$

Taking into account (4.88), we can record equation (4.98) for the slowly changing component in the form

$$Q(p)\theta^{0} + R(p)\kappa_{0}\theta^{0} = S(p)\theta^{0}_{\text{вх}}. \qquad (4.100)$$

Since the equivalent transfer function of the nonlinear element $\kappa_э$ depends on parameters of noise (frequency Ω, amplitude A), all these parameters significantly affect the stability of the system, by which in this case is understood the stability of the equilibrium state of the system, described by equation (4.100). Such stability signifies damping of transient conditions in accordance with the slowly changing component (in our case the useful component θ°).

For investigation of stability of the system the Goldfarb method can be used [101], according to which equation (4.100), with replacement of $p=j\Omega$ can be written as:

$$\frac{Q(j\Omega)}{R(j\Omega)}=-\kappa_э(A,\Omega). \tag{4.101}$$

When (4.101) is used stability is defined as limit of appearance of self-oscillations. Parameters of self-oscillations can be found graphically at the point of crossing of two lines on a plane, one of which the amplitude-phase characteristic of the equivalent linear system $W_л(j\Omega)$ — corresponds to the range of determination of $0<\Omega<\infty$, while second the reverse amplitude characteristic of the nonlinear element

$$W_{обр}=-\frac{1}{\kappa_э(A,\Omega)} \tag{4.102}$$

corresponds to the domain of definition $0<A<\infty$.

This method is convenient during the analysis of stability of a system if only the direction finder is nonlinear and the remaining part of the circuit does not contain nonlinear elements.

We will, with the help of this method, analyze a hypothetical ASN system, the amplitude-phase characteristic of which is depicted in Fig. 4.29 (system open at point K, Fig. 4.23). On the same figure is plotted the reverse characteristic of the nonlinear direction finder $W_{обр}$. The point of crossing A characterizes parameters of oscillations

(in particular, frequency $\Omega_{кр}$). Loss of stability in this case occurs with decrease of transfer function to the value determined at point A.

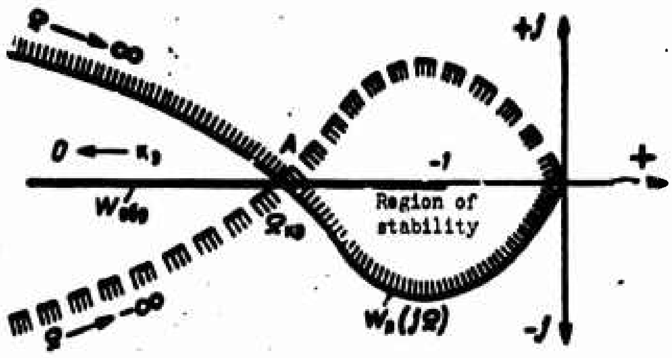

Fig. 4.29. Influence of change of transfer function of direction finder on stability of ASN system.

In the presence, in addition to the direction finder, of other nonlinear elements should be made of stability in accordance with logarithmic amplitude and phase characteristics (see 3.6).

CHAPTER 5

METHODS OF ACTIVE JAMMING OF AUTOMATIC RANGE AND SPEED TRACKING SYSTEMS

5.1. Introduction

Channels of automatic range and speed tracking are present in almost all radar meters of guidance and homing guidance circuits. These channels fulfill the following tasks:

- increase the selectivity of guidance systems, thanks to which it is possible to lock the guidance (homing guidance) circuit on assigned target;
- increase the noise immunity of systems by reducing the time of the open state of receivers and narrowing their passbands;
- act as meters of range and speed coordinates.

The basic assignment of these channels of homing devices is additional selection of targets with respect to range and speed.

5.2. Active Jamming of Automatic Range Finders

For systems of automatic range tracking active interference of two types is created: noise and pulse.

Let us briefly cover the principle of action of the automatic range finder, a block diagram of which is presented in Fig. 5.1. Pulse (C) reflected from target together with interference (П) goes to the input of time discriminator (BP), which is also fed two strobe

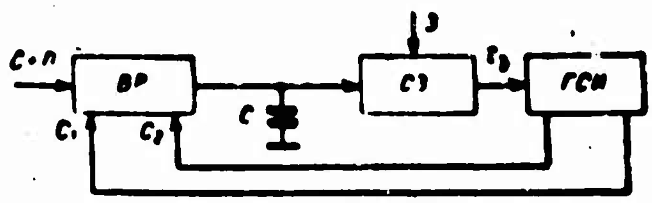

Fig. 5.1. Functional diagram of radio range finder: BP – time discriminator; SZ – delay circuit; 3 – delay voltage; GSN – strobe pulse generator.

pulses C_1 and C_2. The time discriminator constitutes, as a rule, a coincidence circuit, charging and discharging a storage capacitor in dependence upon the sign of the time mismatch ξ between axis of symmetry of reflected and strobe pulses (Fig. 5.2). If the time mismatch $\xi>0$, the voltage on the capacitor is increased; otherwise it decreases.

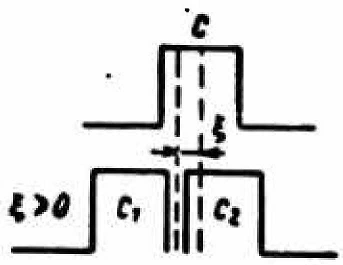

Fig. 5.2. Tracked and strobe pulses in radio range finder: C – pulse reflected from target; C_1 and C_2 – selector pulses.

Dependence of voltage increase Δu_c on capacitor on time mismatch ξ is called the characteristic of the time discriminator and has the form depicted in Fig. 5.3.

Voltage from capacitor u_c influences the delay circuit (SZ), which controls the triggering of the strobe pulse generator (GSI). The delay of strobe pulses is changed in such a way as to reduce the initial mismatch ξ to zero.[1]

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[1]Automatic range tracking systems are discussed in detail in work [36].

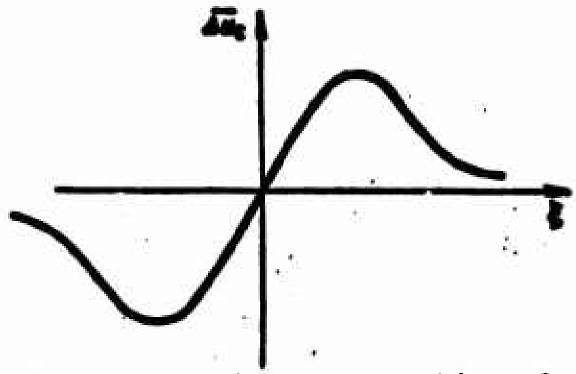

Fig. 5.3. Characteristic of time discriminator.

Noise Jamming

In the presence of noise jamming of high level the automatic finder is acted upon by a combination of pulses with parameters (mainly, repetition period), in consequence of which the value of time shift (delay) of strobe pulses t_3 will be random. In certain measure there is wandering of range gate analogous to that observed in processes of diffusion. Quantitative investigation of the random process of wandering of the range gate can be conducted with the help of the mathematical methods utilized in the theory of Brownian movement.

In a number of theoretical and experimental works it is shown that during the influence on the automaticrange finder a mixture of signal and noise quantities ξ and t_3 also take random values. If the signal-to-noise ratio is sufficiently high, the probability that the range gate will be delayed for a time corresponding to distance to target is for practical purposes, equal to zero, i.e., the finder is driven from tacking conditions. Thus, the automatic range tracking circuit is open.

Confusion Pulse Jamming

Confusion pulse jamming constitutes a sequence of return pulses that are delayed with respect to the signal by τ_3 and monotonically vary from zero to a defined value.

Such interference can be created, for example, by a station built after the block diagram depicted in Fig. 5.4.

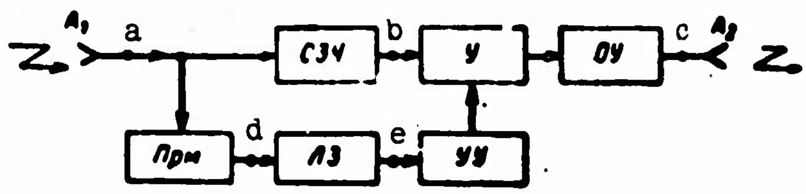

Fig. 5.4. Block diagram of transmitter for range confusion jamming.

The received pulse (Fig. 5.5a; point a in Fig. 5.4) goes to the frequency memorization circuit SZCh, the assignment of which was defined earlier, and the receiver PRM. At the output of the latter are formed video pulses (Fig. 5.5b), which then go to delay line LZ. From the output of the delay line is taken a pulse displaced with respect to the received signal by time $\tau_з$. This pulse is fed to the control unit UU, is strengthened, and then is fed to the high-frequency amplifier **У** opening it for time $\tau_и$, as a result of which at the output of the transmitter there is formed a radio pulse, delayed for time $\tau_з$ relative to the received signal (Fig. 5.5d). The value of delay $\tau_з$ changes with time. The law of change can be different, for example, corresponding to accelerated motion of target (Fig. 5.6).

Let us consider the physical processes connected with the influence of confusion pulse interference on the automatic range finder.

At the initial moment, corresponding to the moment of activation of the jamming transmitter, the delay time of the interference signal $\tau_з=0$, in consequence of which at the input of the automaticrange finder there arrive simultaneously two time-coincident pulses - signal (C) and interference (П) (Fig. 5.7a). At subsequent moments of time there is appearance of time shift (mismatch $\tau_з>0$) of interference pulse with respect to desired signal (5.7b). If the amplitude of the interference pulse is greater than the amplitude of the desired signal, there is

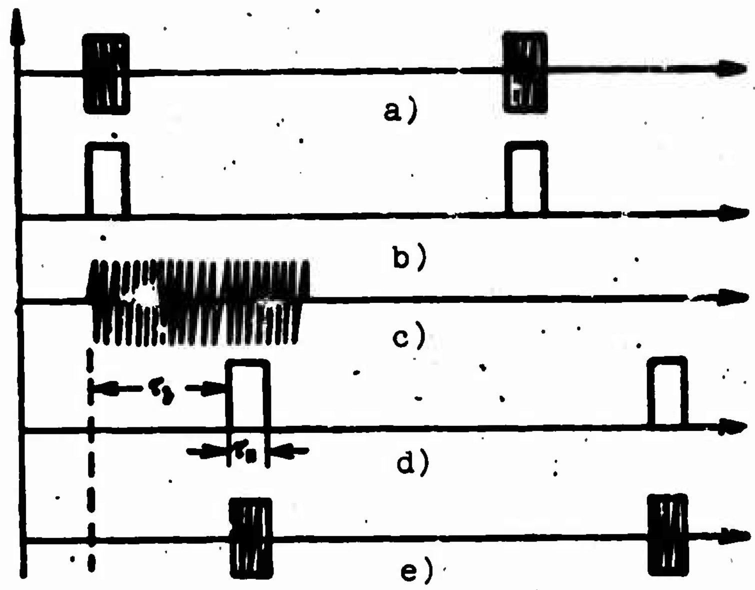

Fig. 5.5. Time diagrams explaining the principle of creation of range confusion interference: a) radar pulse at input of receiver and circuit for memorization of frequency of jamming station; b) pulse at receiver output; c) oscillations at output of frequency memorization circuit; d) pulse at output of delay line; e) interference signal.

Fig. 5.6. Change of value of delay in the case of uniformly accelerated motion of target.

displacement of the range gate (strobe pulses C_1 and C_2) in the direction of a more powerful interference signal П. With further increase of delay τ_3 the range gate "loses" the target, and the finder turns to tracking only the false target simulated by the interference signal (Fig. 5.7c).

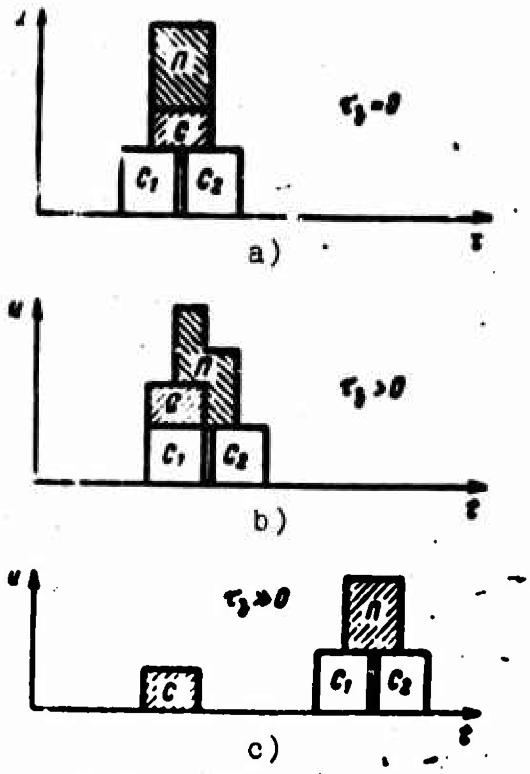

Fig. 5.7. Mutual location of strobe pulses (C_1 and C_2), signal (C), and interference signals (П): a) initial moment, corresponding to moment of switching on jamming transmitter ($\tau_3 = 0$); b) intermediate stage of action of interference ($\tau_3 > 0$); c) tracking of interference signal.

However, in spite of the fact that the automaticrange finder tracks the false target simulated by the interference signal, automatic homing guidance of missile to the real target (carrying the jamming transmitter), in general, cannot be upset. This is explained by the fact that the basic information necessary for homing guidance of the missile, for example, the sighting angle of the target or the angular velocity of the sighting line, is fed to the system of aerodynamic control from the angle-tracking unit, being an element in the radio link of the homing guidance system. The goniometric channel in this case will function by using as the interference signal as working signal. The interference signal, as it is easy to see, carries the information on angular coordinates of the target, which carries the source of interference.

In a number of cases of jamming automatic range finders the guidance (homing guidance) circuit is acted upon directly, for example, for the suppression of radars applied in fire systems for unguided weapons (guns, missiles, and so forth). In these systems errors in range during determination of lead angle are converted directly to angular errors. Errors in range are converted directly to angular errors with certain methods of command guidance of missiles.

5.3. Active Jamming of Automatic Speed Tracking Systems

Automatic tracking of targets with respect to speed makes it possible to select moving targets against a background of passive interference and local objects. Selection of moving targets with respect to speed is realized most simply in radar with continuous on quasi-continuous radiation, using narrow-band signals.

Circuits for automatic speed tracking are based on the principle of frequency filtration of signals reflected from targets moving with different radial velocities relative to a given point of observation [38].

Information on radial target velocity is contained in the value of doppler shift in the frequency of the reflected signal, equal to

$$f_{д}=\frac{2v_r}{c}f_0, \tag{5.1}$$

where v_r – radial component of target velocity;
c – velocity of light in free space;
f_0 – carrier frequency.

This makes it possible, by using narrow-band sounding signals in the radar, to ensure considerable weakening (by tens of decibels) of signals reflected from motionless and slowly moving objects (local objects, clouds of dipole reflectors) and to thus separate fast moving aircraft (targets) on their background.

The method of protecting radar from passive interference by

filtration of frequencies is considerably more effective than the method founded on alternate-period compensation of signals reflected from a cloud of dipoles, applied in pulse radar.

A simplified block diagram of a channel for speed selection for continuous-wave radar, used in the above-mentioned fire-control system, is presented in Fig. 5.8.

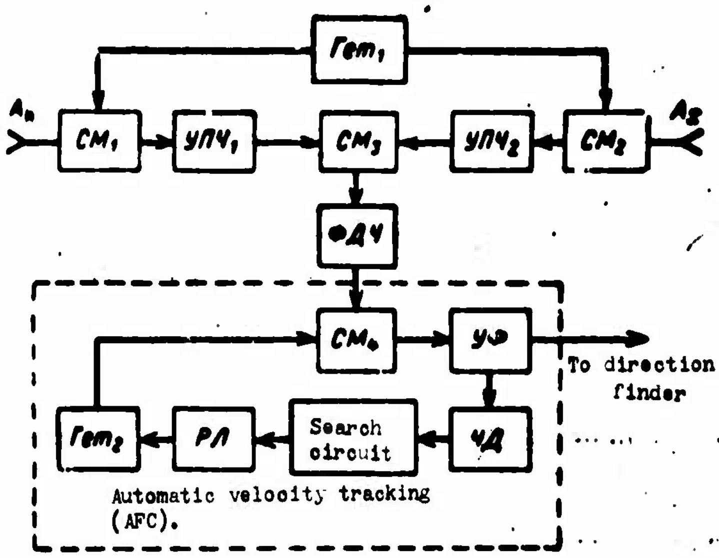

Fig. 5.8. Block diagram of channel of selection of target in speed.

The rear-looking antenna A_X of the missile's homing device takes the signal from the target illumination radar, which is used on the missile as a reference signal. The forward-looking antenna A_H picks up the reflected signal, carrying information on angular position of target and parameters of its motion.

Both signals (reference and reflected) go to corresponding mixers Sm_1 and Sm_2, also fed voltage from the common local oscillator Get_1. As a result, at the output of the mixers are formed signals of intermediate frequency, which are strengthened by corresponding amplifiers $UPCh_1$ and $UPCh_2$.

The input of the third Sm_3 is fed two signals (reflected and reference), the frequencies of which are different and depend on the radial velocity components of motion of target and missile (Fig. 5.9).

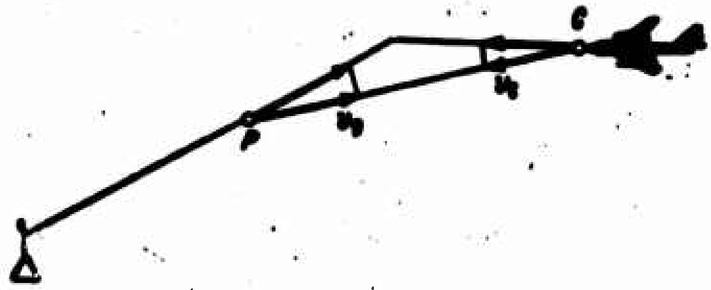

Fig. 5.9. Vector diagram of velocities of guidance circuit.

Let us determine values of doppler frequencies for the simplest homing guidance circuits, when the guided missile and aircraft move directly toward each other (Fig. 5.10).

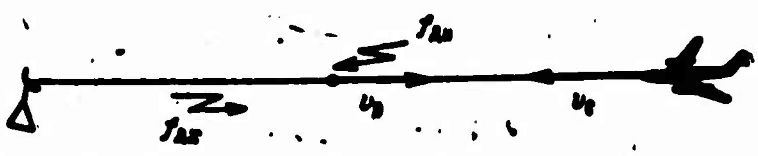

Fig. 5.10. Formation of doppler frequencies of signals picked up by the forward- and rear-looking antennas of a guided missile.

Doppler frequencies of signals $f_{\text{дп}}$ and $f_{\text{дз}}$, picked up by forward- and rear-looking antennas respectively, are equal to

$$f_{\text{дп}} = \frac{2v_c}{c} f_0 + \frac{v_p}{c} f_0 \tag{5.2}$$

$$f_{\text{дз}} = -\frac{v_p}{c} f_0 \tag{5.3}$$

where $\frac{2v_c}{c} f_0$ — component of doppler frequency caused by motion of target aircraft;

$\frac{v_p}{c} f_0$ — component caused by motion of missile;

v_c - radial (with respect to missile) velocity of target aircraft;

v_p - radial (with respect to aircraft) velocity by missile;

c — velocity of light;

f_0 - carrier frequency of suppressed radar.

The difference frequency at the output of the doppler filter FDCh is equal to

$$f_{д}=\frac{2f_0}{c}(v_0+v_p). \qquad (5.4)$$

As a result of the mixing of these signals, at the output of Sm_3 there is a voltage of difference frequency, equal to $f_{д}$, which is filtered by doppler filter FDCh. The bandpass of the FDCh corresponds to the possible range of change of speed of missile and target.

The signal of doppler frequency from the output of the FDCh is fed to the automatic velocity tracking circuit, constituting an ordinary AFC circuit.

Under conditions of doppler frequency search (velocity target search) mixer Sm_4 is fed voltage from oscillator Get_2, the frequency of which is varied in sawtooth fashion by means of reactance tube RL. During the time when the difference frequency

$$\Delta f=f_{д}-f_{г2}, \qquad (5.5)$$

where $f_{д}$ – doppler frequency,

$f_{г2}$ – frequency of oscillator Get_2,

is within the limits of the passband of narrow-band selective filter UF, called the "velocity gate," at its output is formed a signal, which, after passage through frequency discriminator ChD, in form is like the characteristic of the discriminator. The signal from the output of the frequency discriminator is fed to the search-stop circuit, after which there is lock-on of reflected signal and automatic tracking of it with respect to speed (doppler frequency).

Channels for selection of target in speed can be dealt the following forms of jamming:

– narrow-band noise jamming, whose spectrum covers a given range of possible doppler shifts of the frequencies of the reflected signal;

– confusion jamming, created by simulation of false doppler frequencies.

Noise Jamming

During the influence on speed selection circuit of an additive mixture of signal and sufficiently intense interference in the form of white noise, the voltage at the input of the frequency discriminator (ChD) can be represented as a quasi-harmonic oscillation with random amplitude U(t) and phases $\psi(t)$:

$$u = U(t) \cos [\omega_0 t + \psi(t)],$$

where U(t) and $\psi(t)$ are random functions of time.

The voltage at the output of the frequency discriminator will also be a random function of time, and consequently the parameters of the reactance tube will also change in random fashion. Accordingly, the frequency of the local oscillator is subject to random changes. Owing to the closed-loop nature of the automatic control system, the random frequency shifts of the local oscillator, in turn, will cause random changes of frequency difference $\Delta f_{\text{гд}} = f_{\text{д}} - f_{\text{г}}$. Sometimes to characterize these frequency shifts of the oscillator in the closed-loop tracking system, the terms "random wandering" or simply "wandering," borrowed from diffusion theory, are used. We often talk about "wandering of the velocity gate," although the "velocity gate" can not actually wander, inasmuch as it is a narrow-band filter with fixed tuning.

Under the influence of the "velocity gate" noise is displaced along its frequency axis, and upon the expiration of a certain amount of time, commensurable with the time constant of the automatic velocity tracking channel, the doppler frequency of the signal will exceed the bandpass of the frequency discriminator (the aperture of the discrimination characteristic), as a result of which autotracking circuit loses the target. If in the homing device provision is not

made for conditions of approach to a source of active interference disturbance of the operation of the speed selection circuit will lead to full breaking of the guidance circuit.

During the creation of noise jamming to channels of selection with respect to speed rather stringent requirements are imposed on the band of radiated noises. The bandwidth of noises ΔF_n is determined from the condition of ensuring coverage of doppler frequencies of all useful signals from the covered aircraft of a group in different combat formations and different attitudes relative to the suppressed homing device.

Confusion Jamming

Confusion jamming of channels of velocity selection decoy the "velocity gate" and stop the doppler frequency tracking of the desired signal.

The possibility of decoying the "velocity gate" is based on the peculiarities of influence of two signals (useful and interference) with different amplitudes and frequencies on the frequency discriminator.

During influence on the frequency discriminator of two harmonic oscillations with fixed frequency the system of automatic velocity tracking (ASS) is most fully described by a family of generalized discrimination characteristics, by which is understood dependence of output voltage of the frequency discriminator on the shift of one of the signals relative to the transition frequency ω_0.[1] The parameter of the family usually is the difference in frequencies of the two signals $\Delta\omega = \omega_1 - \omega_2$.

Let us find the generalized discrimination characteristic for the frequency discriminator with upset circuits (Fig. 5.11). There

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[1]The transition frequency ω_0 is that to which corresponds zero output voltage of the frequency discriminator.

is analogy here with the case of influence of two signals on the ASN system (4.3).

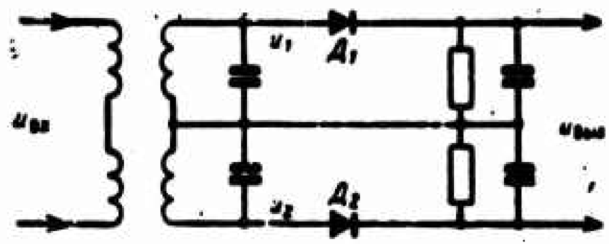

Fig. 5.11. Frequency discriminator with upset circuits.

Let us assume that the resonance characteristics of both circuits of the discriminator are even (Fig. 5.12) and are determined by formulas

$$g(\Delta\omega_p - \Delta\omega) = \frac{1}{\sqrt{1+(\alpha_p-\alpha)^2}}, \tag{5.6}$$

$$g(\Delta\omega_p + \Delta\omega) = \frac{1}{\sqrt{1+(\alpha_p+\alpha)^2}}, \tag{5.7}$$

where

$$\alpha_p = \frac{2\Delta\omega_p}{\Delta\omega_{0.7}}; \quad \alpha = \frac{2\Delta\omega'}{\Delta\omega_{0.7}};$$

$\Delta\omega_{0.7}$ – bandwidth of the circuit at the 0.7 level; $\Delta\omega_p$ – detuning of circuits of the frequency discriminator relative to transition frequency ω_0; $\Delta\omega'$ – deviation of frequency of input signal from nominal value ω_0.

For the voltage at the discriminator input it is possible to record

$$u_{вх} = U_п \cos\omega_1 t + U_с \cos\omega_2 t.$$

At the input of detectors $Д_1$ and $Д_2$, which we will consider square-law, we obtain

$$u_1 = \kappa_д [U_п g(\Delta\omega_p - \Delta\omega_1)\cos(\omega_1 t + \psi_1) + U_c g(\Delta\omega_p - \Delta\omega_2)\cos(\omega_2 t + \psi_2)],$$
$$u_2 = \kappa_д [U_п g(\Delta\omega_p + \Delta\omega_1)\cos(\omega_1 t + \psi'_1) + U_c g(\Delta\omega_p + \Delta\omega_2)\cos(\omega_2 t + \psi'_2)],$$

$\Delta\omega_1$, $\Delta\omega_2$ – deviations of frequencies of interference and signal from transition frequency ω_0;
ψ_1, ψ_2, ψ'_1 ψ'_2 – high-frequency phase shifts.

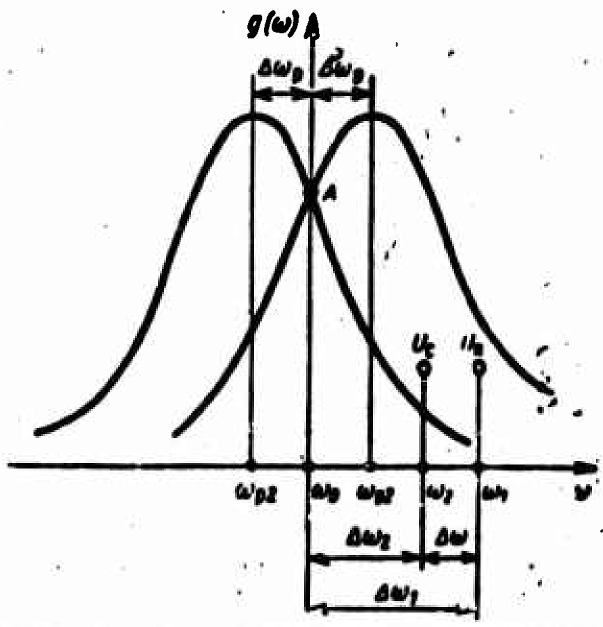

Fig. 5.12. Resonance characteristics of discriminator circuits.

After detection and filtration of signal by the r-c output filter, for output voltage we obtain

$$u_{вых} = \kappa U_c^2 \{b^2 [g^2(\Delta\omega_p - \Delta\omega_1) - g^2(\Delta\omega_p + \Delta\omega_1)] + g^2(\Delta\omega_p - \Delta\omega_1 + \Delta\omega) - g^2(\Delta\omega_p + \Delta\omega_1 - \Delta\omega)\}, \tag{5.8}$$

where κ is a constant; $b = \frac{U_п}{U_c}$.

Relationship (5.8) determines the generalized discrimination characteristic. Comparing (5.8) with (4.21), we note their coincidence. This makes it possible to use results obtained in the case of

of noise influences on goniometric channels.

If on the ASS system is acted upon only by a signal with frequency ω_1, the zero of the discrimination characteristic (point 0) in steady-state operation coincides with frequency ω_1 (curve 1 in Fig. 5.13). During the influence of a signal with frequency ω_2 at the discriminator output there is error voltage of positive sign, which means that the point of the stable state of equilibrium of the system ("stable zero" of the generalized discrimination characteristic) starts to shift to the right, and upon the expiration of transient conditions the zero of the discrimination characteristic will be at a certain point 0', lying between values of frequencies ω_1 and ω_2 (curve 2 in Fig. 5.13).

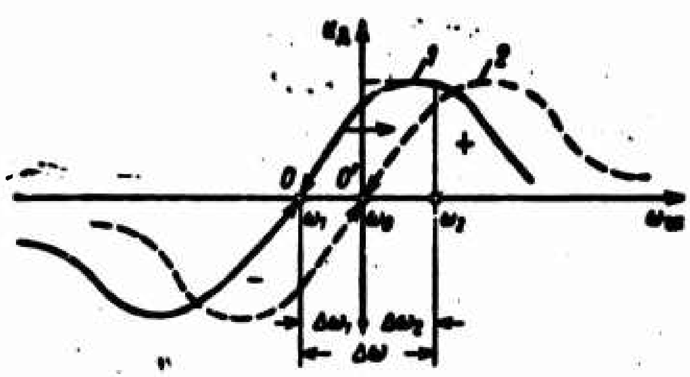

Fig. 5.13. Discrimination characteristics of automatic velocity tracking system (ASS): curve 1 – for the case of action of one signal with frequency ω_1; curve 2 – for the case of action of two signals with frequencies ω_1 and ω_2.

Position of point 0' determines tracking error $\Delta\omega_2$ for signal frequency ω_2. In general error $\Delta\omega_2$ is found by solving of equation (5.8).

$$b^2[g^2(\Delta\omega_p-\Delta\omega_1)-g^2(\Delta\omega_p+\Delta\omega_1)]^2+ \\ +g^2(\Delta\omega_p-\Delta\omega_1+\Delta\omega)-g^2(\Delta\omega_p+\Delta\omega_1-\Delta\omega)^2=0.$$

In linear treatment during linearization resonance curves at point A (Fig. 5.12) for error $\Delta\omega_2$ we obtain

$$\Delta\omega_2 = \Delta\omega \frac{b^2}{1+b^2}. \qquad (5.9)$$

From (5.9) it follows that within limits of the linear section of the discrimination characteristic point of the stable state 0' will be between frequencies of signal and interference ω_1 and ω_2, being displaced with change of ratio b to the frequency of the source with greater power.

With a sufficiently high signal-to-noise ration the zero of the discrimination characteristic (and consequently the "velocity gate") will track the carrier frequency of the interference signal. If detuning $\Delta\omega$ is increased at a certain rate, the "velocity gate" can be decoyed from the desired signal by a sufficiently great distance along the frequency axis (Fig. 5.14).

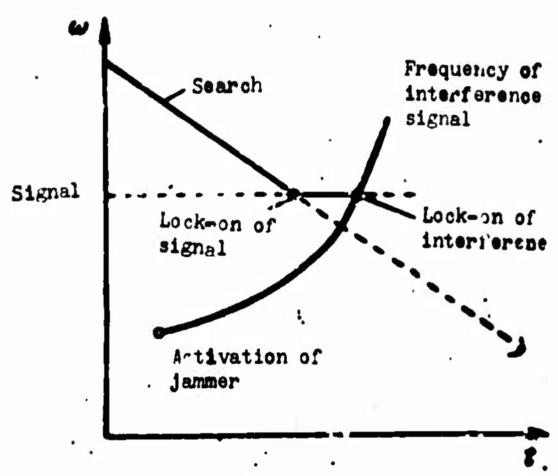

Fig. 5.14. Time diagrams, explaining the principle of action of interference confusing the "velocity gate."

5.4. Possible Methods of Shifting Signals in Carrier Frequency

The shifting of signals in carrier frequency usually is done by means of high frequency phase shifters (tw tubes, magnetized ferrites,

and others) [51, 52].

We will briefly explain the principle of shifting frequency by means of tw tube.

The helix of the tw tube (Fig. 5.15) is fed linearly time-varying voltage

$$u(t)=\varkappa t. \tag{5.10}$$

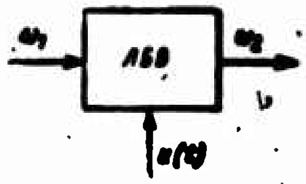

Fig. 5.15. Diagram of frequency shift in a tw tube.

If, for example, this voltage increases linearly, the speed of the electron stream will increase accordingly. This increase of speed of the electron stream (in linear treatment during small changes of voltage) ensures increase of speed of the electromagnetic wave propogated in the "helix-electron stream" system. Owing to the increase in speed of the electromagnetic wave, the phase shift of oscillation in the section of interaction (delay system) will decrease (since the length of the section in which phase shift occurs is fixed, and the propagation velocity of the wave is increased). In Fig. 5.16 is represented approximate dependence of phase shift $\Delta\psi$ on the voltage applied to the helix u.

If change voltage u(t) in linear fashion with positive derivative, phase shift, as a function of time, is changed linearly, but now with negative derivative (Fig. 5.16).

Since the phase shift $\Delta\psi$ changes linearly with time, the phase of high-frequency oscillations at the output of the tw tube can be represented as follows:

$$\varphi=\omega_1 t+(\Delta\psi_0-\kappa' t). \quad (5.11)$$

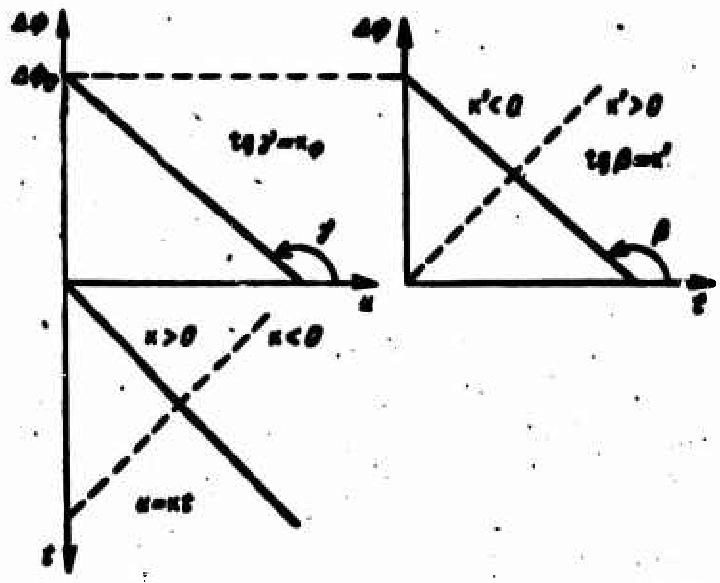

Fig. 5.16. Dependence of phase shift of signal on helix voltage of tw tube.

Here κ' is proportionality factor (slope of line $\Delta\psi = \Delta\psi_0 - \kappa' t$ in system of coordinates $\Delta\psi$, t).

Accordingly, the frequency of oscillations at the output of the tw tube is equal to

$$\omega_2=\frac{d\varphi}{dt}=\omega_1-\kappa', \quad (5.12)$$

i.e., differs from input frequency ω_1 by $\Delta\omega$, where

$$\Delta\omega=\omega_2-\omega_1=\kappa'=\text{const}.$$

Thus linear change of phase leads to shift of frequency of output signal by a certain constant amount $\Delta\omega$.

In practice continuous change of control voltage u(t) with time per linear law is possible only within certain comparatively narrow

limits. Therefore in order to shift frequency it is necessary to apply different forms periodic phase modulation of the tw tube [51].

One of the simplest laws of phase modulation of the tw tube is the sawtooth law (Fig. 5.17). Let us determine the parameters of sawtooth control voltage ($T_{ш}$, κ_1, Δu_0), ensuring the obtaining at the output of the tw tube of a constant positive shift of frequency $\Delta\omega$.

In the case of modulation by sawtooth voltage the phase of the signal at the output of the tw tube can be recorded as follows:

$$\varphi = \omega_1 t + \kappa_\phi\left(\Delta u_0 + \kappa_1 \frac{t}{T_ш}\right), \tag{5.13}$$

if $0 < t < T_ш$;

$$\varphi = \omega_1 t + \kappa_\phi \Delta u_0 = \omega_1 t + \Delta\varphi_0,$$

if $t = T_ш$.

Here $T_ш$ – period of sawtooth voltage (Fig. 5.17);

κ_ϕ – slope of modulation characteristic of tw tube (Fig. 5.16);

κ_1 – proportionality factor.

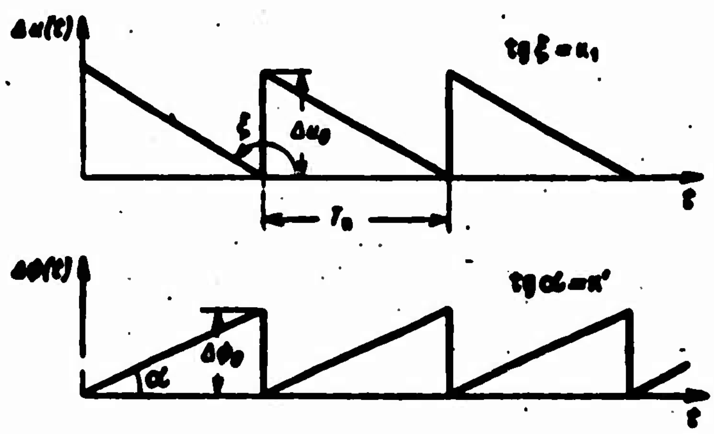

Fig. 5.17. Diagrams explaining frequency shift in tw tube with the help of sawtooth voltage. Δu(t) and Δψ(t) characterize the law of change with time of increases in modulating voltage and phase of oscillations at output of tw tube respectively.

Equation (5.13) assumes linear dependence of phase shift of high-frequency oscillations at output of tw tube on modulating voltage

$$\Delta\phi = K_\phi \Delta u. \qquad (5.14)$$

Differentiating ϕ with respect to time, we find the value of frequency at the output of the tw tube

$$\omega_2 = \omega_1 + K_\phi K_1 \frac{1}{T_n}. \qquad (5.15)$$

Thus, the sought frequency shift equals

$$\Delta\omega = K_\phi K_1 \frac{1}{T_n} = \frac{\Delta\phi_0}{T_n}. \qquad (5.16)$$

Since modulation is carried out by sawtooth voltage, to shift frequency ω_1 by $\Delta\omega$ without disturbing the monochromaticity of oscillations, in principle, is possible only in that case when the period of the sawtooth modulating voltage T_n is multiple of $T = \frac{2\pi}{\Delta\omega}$, i.e.,

$$T_n = nT, \qquad (5.17)$$

where n = 1, 2, 3, ..., and return period of the saw is equal to zero.

In other words, in every period of the modulating saw T_n must contain a whole number of segments with duration T. This can be clarified with help of the vector diagram shown in Fig. 5.18a. The unmodulated input signal with frequency ω_1 can be represented in the form of vector $\bar{u}(t)$, revolving with constant angular velocity ω_1. Linear phase modulation increases (or decreases) the angular velocity of rotation of this vector by a constant amount $\Delta\omega$. Modulation by periodic sawtooth voltage will be equivalent to linear modulation if during the time equal to the period of the modulating sawtooth voltage vector $\bar{u}(t)$ additionally makes a whole number of rotations

$$n_{\text{дол}} = \frac{2\pi T_{\text{п}}}{\Delta\omega}.$$

If the return period of the saw is close to zero, oscillations of frequency ω_2, corresponding to adjacent periods of the sawtooth modulating voltage, will continuously, without a jump in phase, repeat one another (Fig. 5.18b and c). When conditions of multiplicity of $T_{\text{п}}$ and T are not met, oscillations of frequency ω_2, corresponding to adjacent sections of modulating saw, will have different phases, which will lead to disturbance of monochromaticity of oscillations at the output of the tw tube (Fig. 5.18d and e).

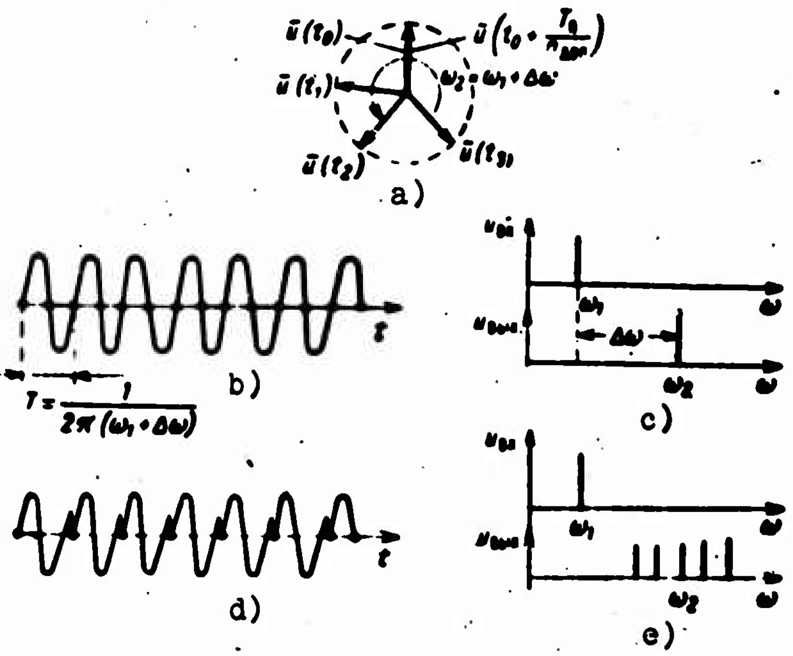

Fig. 5.18. Principle of shifting of frequency by tw tube: a) vector diagram of harmonic oscillations in tw tube; b, c) output voltage of tw tube and its spectrum with correctly selected parameters of control voltage; d, e) output voltage of tw tube and its spectrum when parameters of control voltage are selected incorrectly.

Inasmuch as the slope of the saw and its period are determined completely by the necessary value of frequency shift $\Delta\omega$, the same determines the amplitude of the sawtooth voltage Δu_0. Actually, in view of (5.16) and (5.17) and also considering that $T_{\text{п}}$ is a multiple

of T, ψ_0 must in the same measure be a multiple of 2π. Hence in accordance with (5.14) it follows that

$$\Delta\omega_0=\frac{2\pi n}{\tau_\phi}. \quad (5.18)$$

where $n = 1, 2, 3...$

In practice with linear phase modulation of tw tube by sawtooth voltage at the output of the tw tube there appear oscillations at two frequencies, shifted in opposite directions relative to input frequency ω_1. One of these oscillations corresponds to forward movement of the saw, the second to reverse movement.

For ferrite phase shifters, in contrast to tw tubes, to the positive slope of the "saw" of control current i(t) also corresponds a positive shift of frequency of output signal (Fig. 5.19a), and to the negative slope corresponds a shift of frequency in the direction of smaller values (Fig. 5.19b).

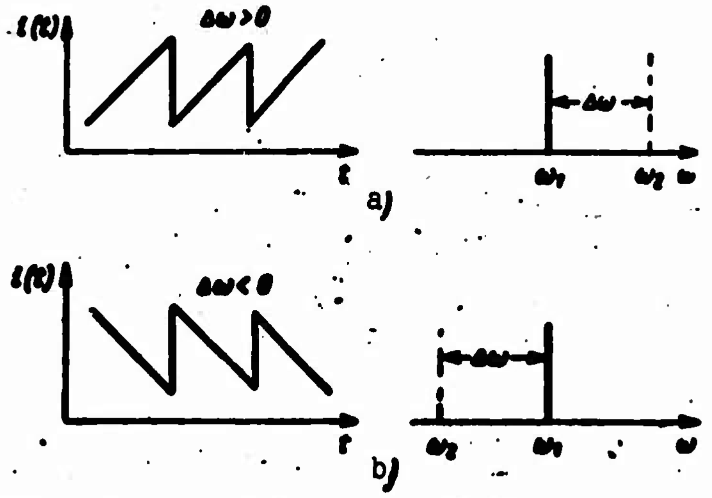

Fig. 5.19. Principle of frequency shift with the help of ferrite phase shifters: a) law of change of control current for positive shift frequency; b) law of change of control current for negative shift.

To constant slope of sawtooth control voltage or current corresponds a constant shift of frequency of output signal.

For creation of confusion jamming the frequency of the output signal (and consequently the value of $\Delta\omega$) must be changed with time (decreased or increased) per defined law. Let us find the law of change of control voltage (current) ensuring linear change with time of $\Delta\omega$ and accordingly of ω_2. Assigning the necessary law of change of $\Delta\omega$ in the form

$$\Delta\omega = \kappa_\omega \Delta\omega_0 t, \tag{5.19}$$

by integration of $\Delta\omega$ we determine the sought law of change of phase

$$\Delta\varphi(t) = \int_0^t \Delta\omega\, dt = \frac{\Delta\omega_0 \kappa_\omega}{2} t^2. \tag{5.20}$$

From (5.14) and (5.20), taking into account the integration constant we find control voltage (current)

$$u(t) = u_0 \pm \kappa'' t^2, \tag{5.21}$$

where

$$\kappa'' = \frac{\Delta\omega_0 \kappa_\omega}{2\kappa_\varphi}.$$

Expression (5.21) shows that for frequency shift per linear law the control voltage (current) must be changed not per linear, but parabolic, law.